Mathematical raytracing – outline

- Motivation
  - Connection with oblique ray method
- Organizing system information in a spreadsheet
  - Required information (just like graphical methods)
  - Surface power
  - Reduced thickness
- Raytracing through a thin-lens system
  - The algorithm
  - Ray-height solves
- Raytracing in a more general system
  - The formulas
  - Raytracing using an EXCEL spreadsheet
- More complex aspects
  - Stops, pupils, other types of solves, backwards ray tracing
  - Viewgraphs included, but we won't cover it in class

ynu raytracing-motivation

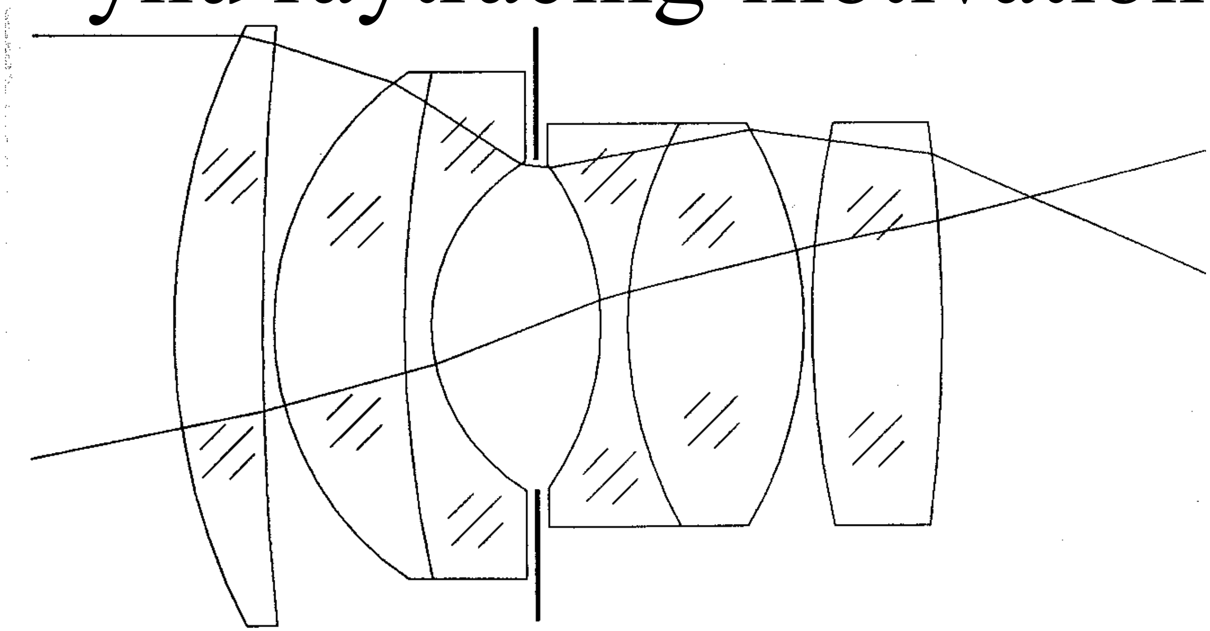
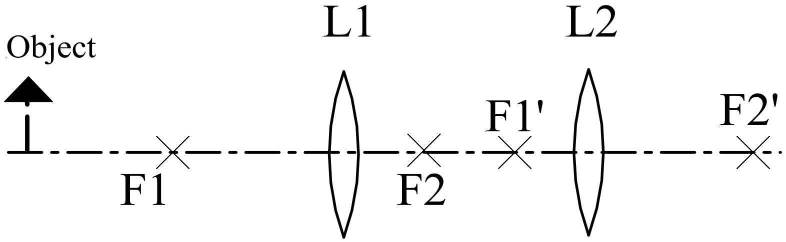


Figure 12.2.- Double Gauss lens

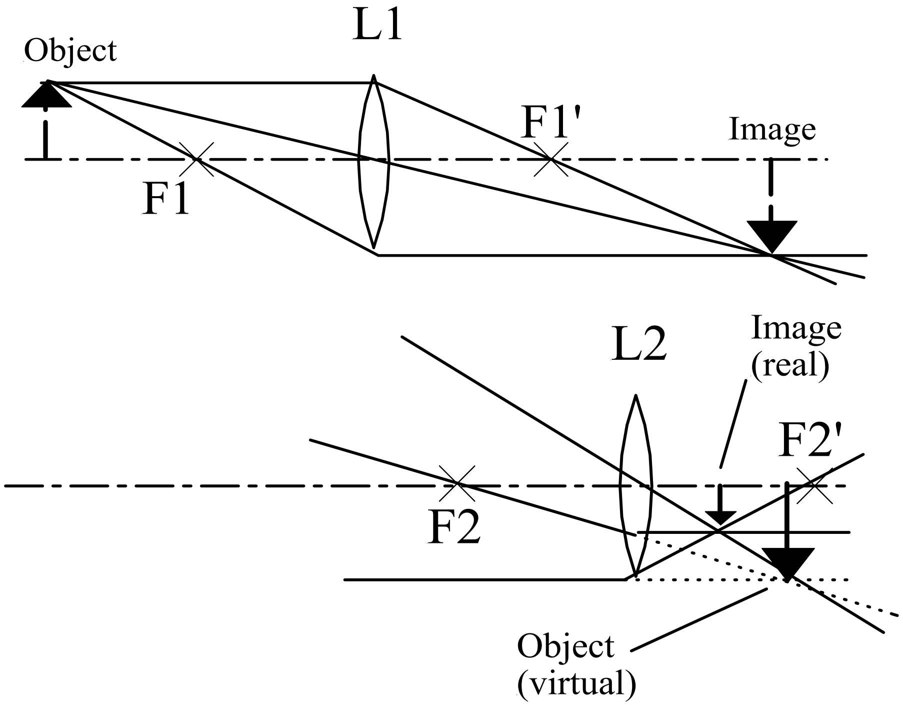
- Even for a two-lens system, the methods we have learned so far are unwieldy and can easily lead to confusion
- Each problem requires considerable thought about setup and solution method
- A more systematic way of handling complex problems is needed

Sequential raytracing vs. sequential imaging

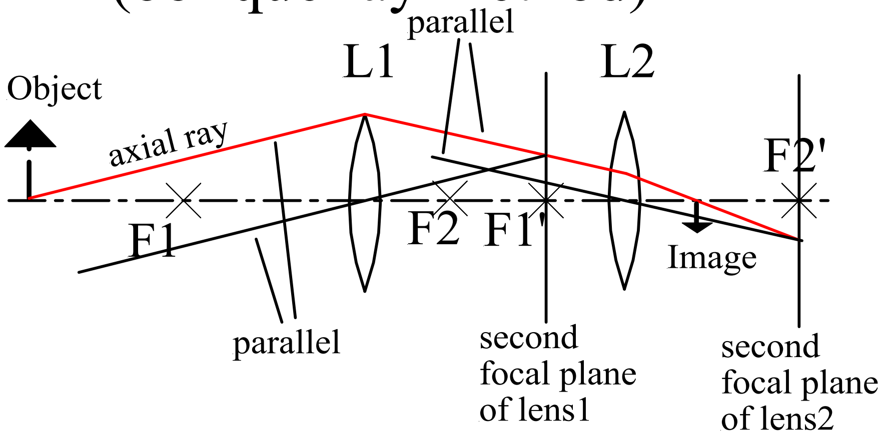


Multiple lens system

Solution by imaging (parallel ray method)



Solution by raytracing (oblique ray method)



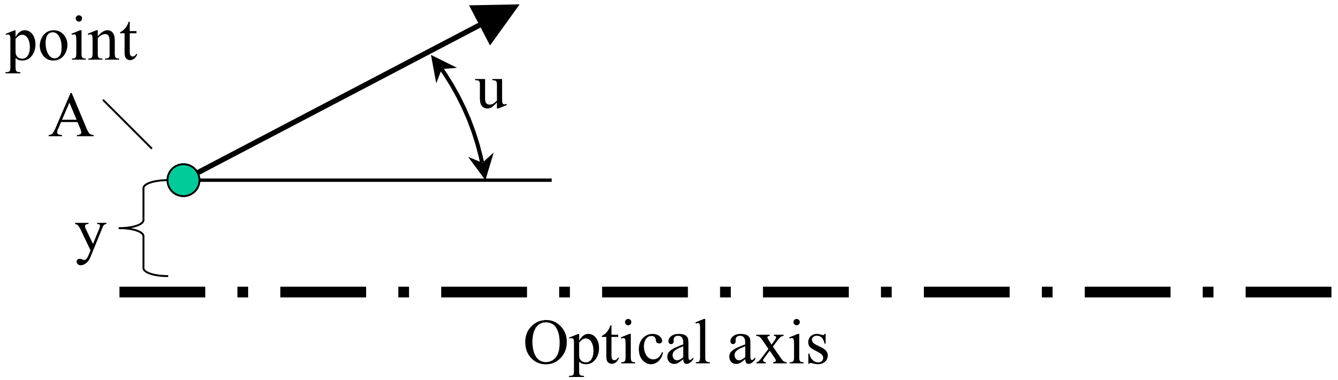
- Both methods can be done mathematically
- Raytracing is easier and more easily automated

ynu vs matrix method

Why do I teach the ynu method rather than matrix method?

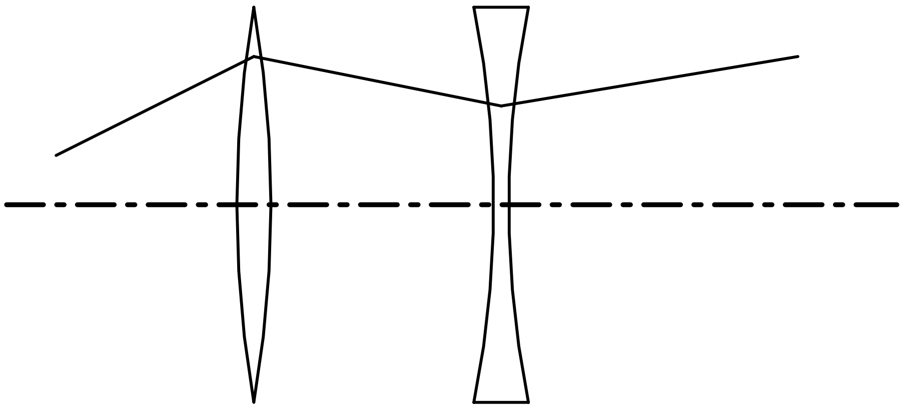
- Matrix method solves same problem
  - The "ABCD" matrix can be obtained from ynu raytrace
    - Anything you can do with matrices you can do with ynu
    - Laser resonator stability often done by ABCD matrices is just as easily done using ynu raytracing
  - Other methods exist also
- Practical reasons
  - Matrix arithmetic would be a big hurdle for some students
  - The method in the book doesn't even show how to find images
    - matrix inverse required
- Other reasons
  - ynu method is a good introduction to commercial raytrace programs
  - ynu method requires less computation

ynu raytracing-notation for ray



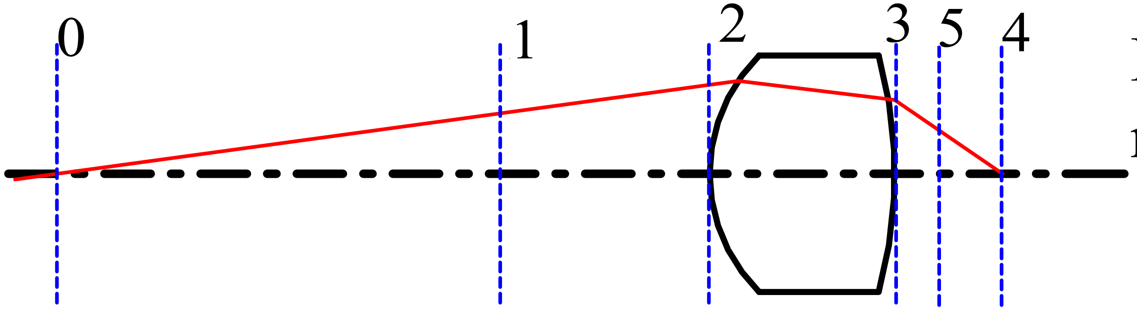
- A ray at a point is given by its height from the optical axis, y, and its slope (tangent of angle), u
  - For small angles u can be angle in radians
  - The diagram represents the height and direction of the ray at point "A"
- Sign conventions
  - Sign conventions are absolutely essential to getting the correct answer
  - y positive if above axis
  - u positive if rotated counterclockwise from axis

Behavior of ray through thin-lens system



- Ray slope (angle) stays constant between lenses
  - Height changes
- Ray height doesn't change at a lens
  - Slope changes

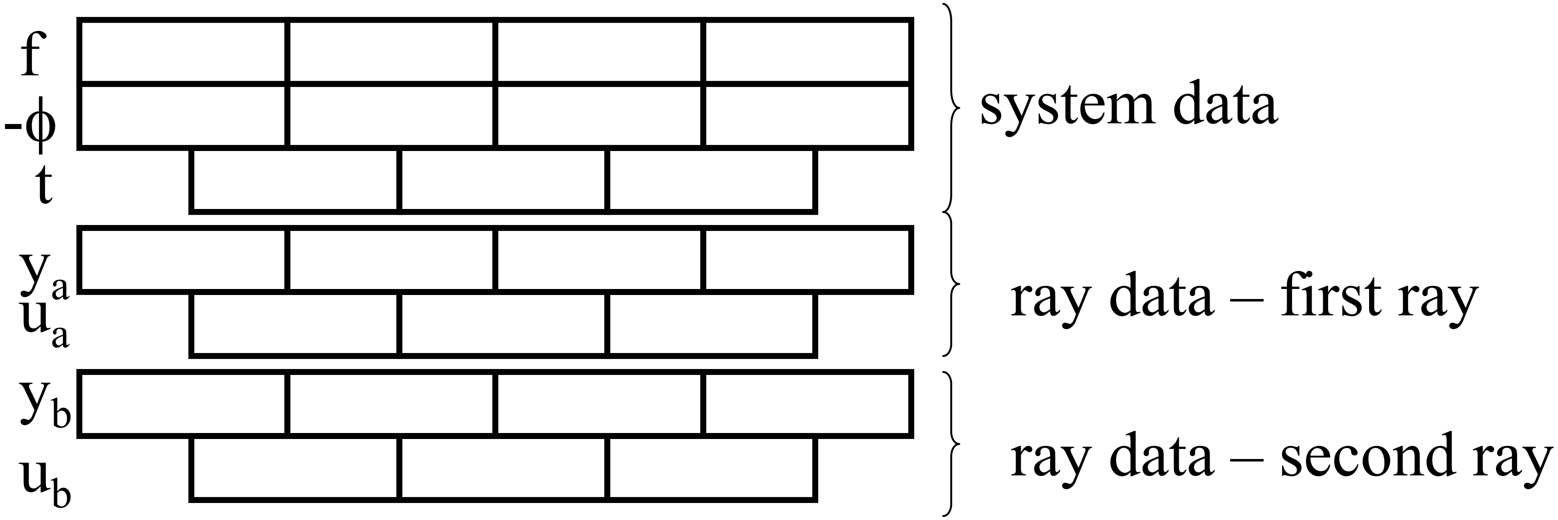
ynu method - surfaces



Dotted blue lines represent surfaces

- A “surface” is any plane at which you want to know the ray height or angle
  - Object surface (0), often numbered “0”
  - Image surface (4)
  - Refracting surface (2 and 3)
  - Dummy surfaces (1 and 5)
    - These could be aperture locations or for some other purpose
  - A thin lens can be considered a surface (since it is thin)
- Surfaces are numbered sequentially in the order in which they are to be calculated
  - Not necessarily from left to right (go backwards from 4 to 5)

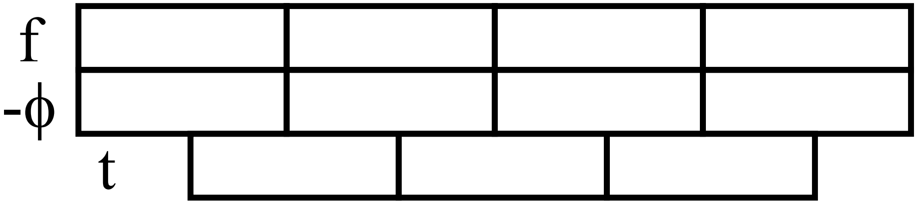
Ynu worksheet – thin lenses



- Offset boxes indicate the spacings between lenses
- y and u stand for ray height and angle, n is for index and why it appears here will become more clear later
- More rays can be traced if desired

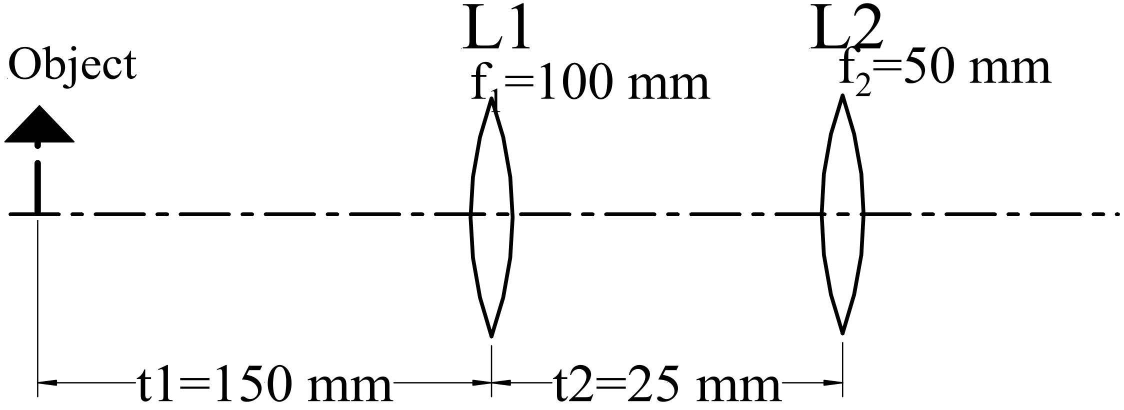
Thin lens ynu raytracing – system data

- First row, lens focal lengths
  - Usually a given, just enter it
- Second row, negative power of lens
  - Take inverse (1/x on calculator) of row 1 and then take negative
    - Careful of double negatives, negative power is positive for a negative lens
  - Object, image and any dummy surfaces have zero power
    - leave focal length blank, enter zero for negative power
- Third row, distances between surfaces
  - Object distance, image distance, distances between lenses



System information part of ynu worksheet for thin lenses

thin lens ynu raytracing – example



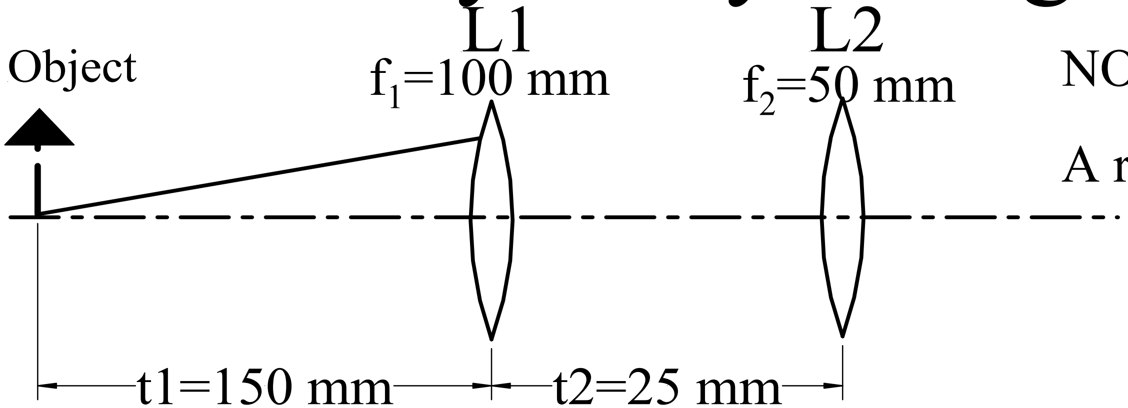
Making a rough sketch of the system before starting is strongly recommended

- Step 1, enter input data for optical system
  - Arrange the boxes to enter numbers so thicknesses fall between the boxes for the lenses, object, or image
  - Units on the raytrace worksheet would make it messy, better to just work in a single unit (mm recommended)

	object		L1		L2		image
f	N/A		100		50		N/A
-ϕ	0		-0.01		-0.02		0
t		150		25		TBD	

The final thickness is the distance to the image. It can't be entered yet as it is not known

thin lens ynu raytracing– ray starting data



NOTE: Drawing not to scale!

A rough sketch is usually sufficient

- Step 2, enter starting data for the ray to be traced

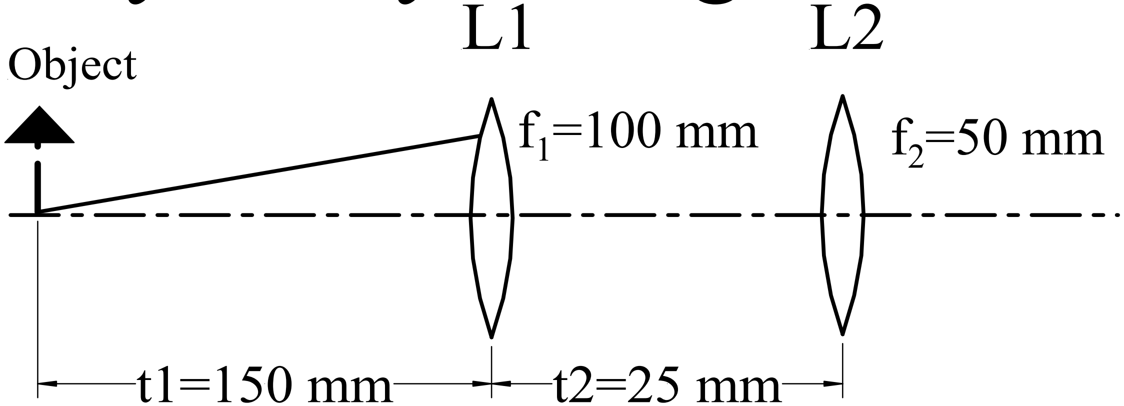
-ϕ			-0.01		-0.02	
t		150		25		
y	0					
u		0.1				

System data and ray data brought together; note how they line up

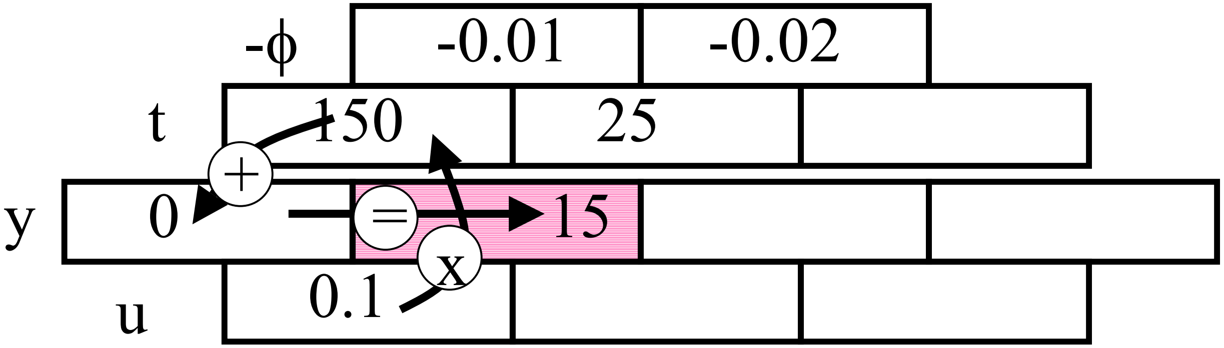
To find image location trace a ray starting at base of object, height=0

Any angle can be put in for u, location of image can be found from any ray starting at base of object

ynu raytracing between lenses



- Step 3, trace ray to first lens
  - Only the height changes



$-\phi$			-0.01		-0.02		
t		150		25			
y	0		15				
u		0.1					

At each step:

a. Start at last filled box

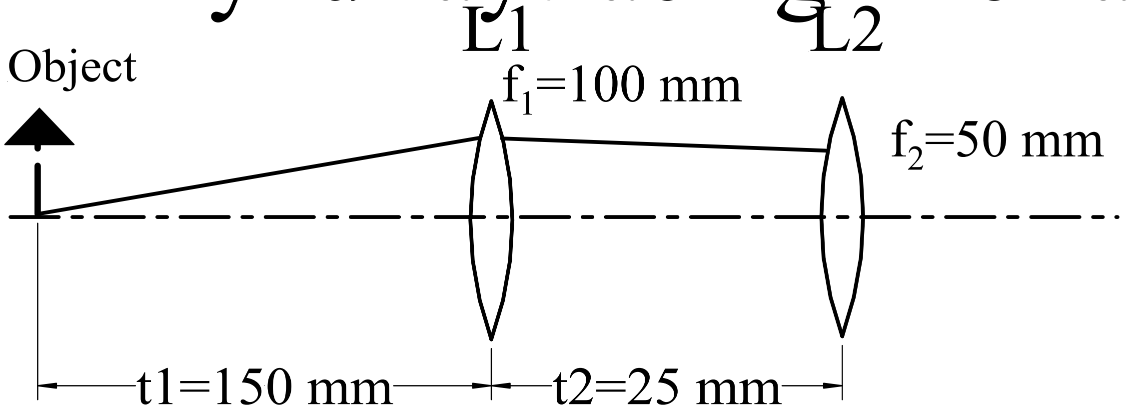
b. multiply by number directly above

c. add to number at left

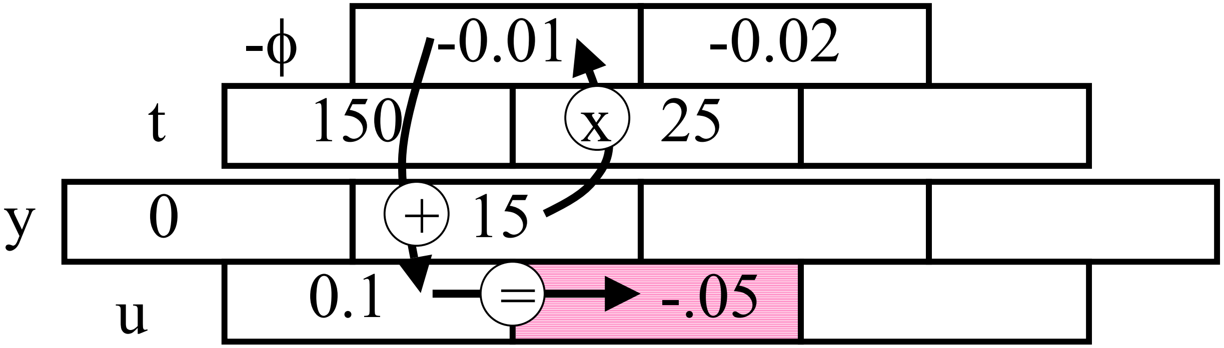
d. Enter result in box to right

Note that the row for focal length has been left off here. It isn't needed in raytrace

ynu raytracing – refraction at lens



- Step 4, refraction at first lens, angle change only
  - Mathematical procedure is identical to previous step



-ϕ	-0.01	-0.02	
t	150	25	
y	0	15	
u	0.1	-.05	

At each step:

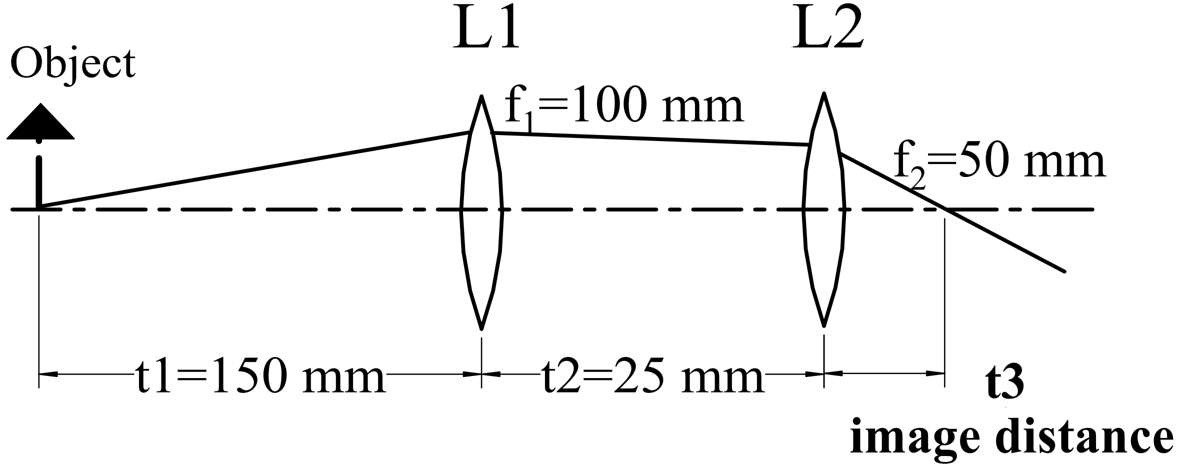
a. Start at last filled box

b. multiply by number directly above

c. add to number at left

d. Enter result in box to right

ynu raytracing – continuing the calculation



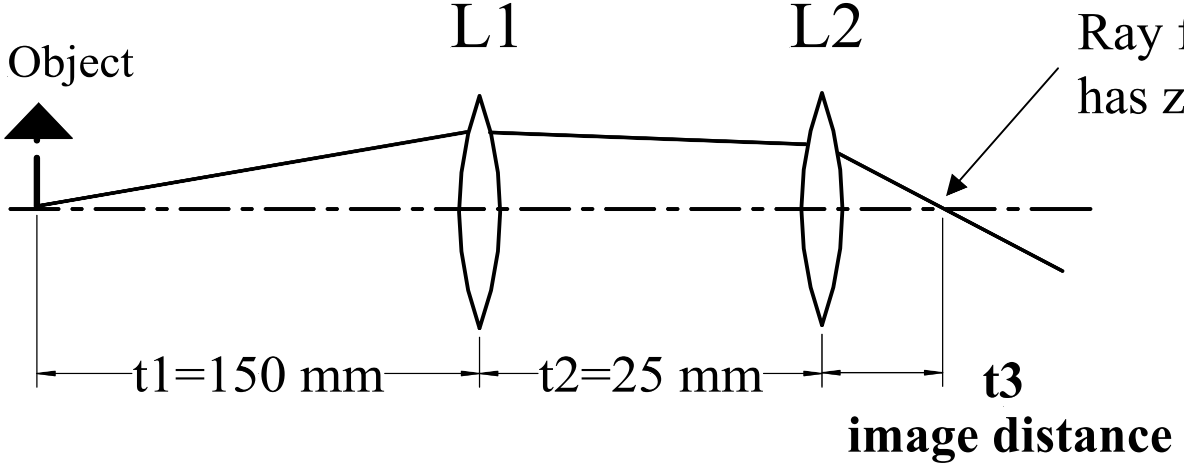
- Step 5, continue through all the lenses in the system
  - Mathematical procedure is identical to previous step

$-\phi$			-0.01		-0.02		
t		150		25			
y	0		15		13.75		
nu		0.1		-.05		-0.325	

We can’t continue because no thickness is given

The next thickness is the image distance which we are trying to find!!!

ynu raytracing – finding the image



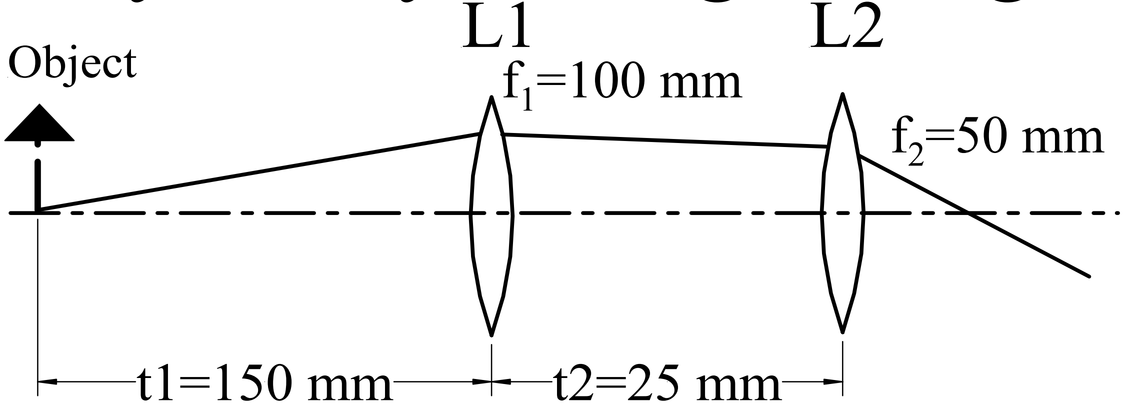
Ray from base of object has zero height at image

- The thickness is not known, but the ray height must be zero at the image, so enter that

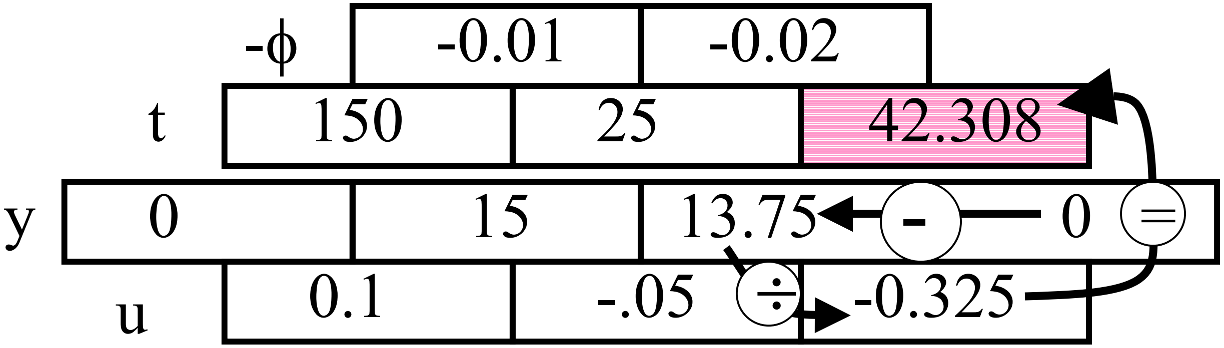
-ϕ			-0.01		-0.02		
t		150		25			
y	0		15		13.75		0
u		0.1		-.05		-0.325	

In this case we know the ray height but not the thickness. We use a <u>height solve</u> in this case (a solution for the thickness which gives the desired ray height)

ynu raytracing – height solves



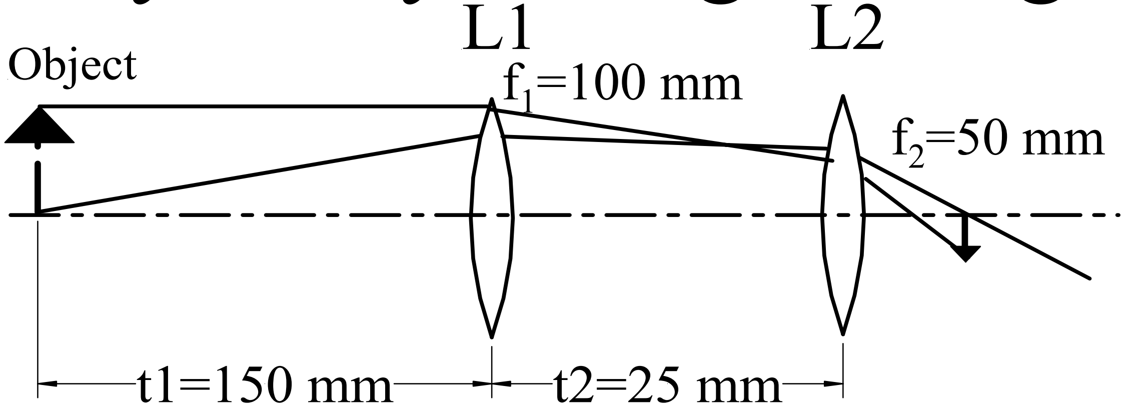
- Either do the algebra yourself or follow the prescription below



-ϕ		-0.01	-0.02	
t	150	25	42.308	
y	0	15	13.75	0
u	0.1	-.05	-0.325	

Note: once you’ve done the solve, you can check your answer using the ordinary procedure

ynu raytracing – image height



- A second ray can be traced just like the first

$-\phi$			-0.01		-0.02		
t		150		25		42.308	
y	0		15		13.75		0
u		0.1		-.05		-0.325	
y	10		10		7.5		-3.077
u		0		-.1		-0.25	

The second ray here starts at the top of the object

The angle doesn’t matter, just pick something convenient. Every ray from top of object goes through top of image

The image distance found previously is used just like any other distance

Ray tracing formulas

- Formula for tracing between lenses
  - y is starting height, y' height after going through the thickness, t
- Formula for refraction at a lens
  - ϕ is power of lens (1/f), y is height at lens (both before and after refraction
  - u', n' after refraction, u, n before refraction
  - For the problems we've considered so far $n'=n=1$

$$y'=y+ut$$

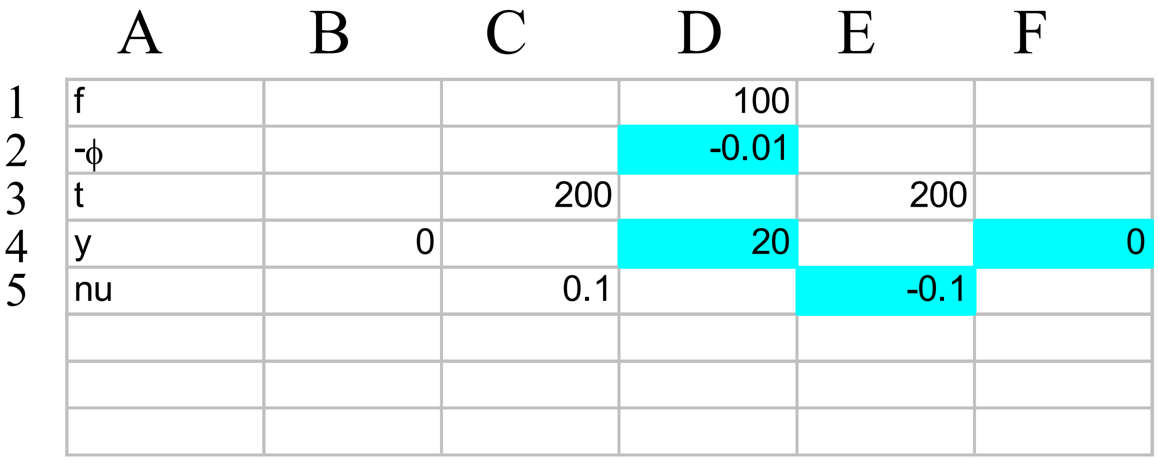
or

$$y'=y+(nu)*(t/n)$$

$$n'u'=nu-\phi y$$

Ray tracing on a spreadsheet

- Each cell has a name
  - A1 is upper left
- Cells can contain different types of data
  - Text, number, formula
  - Light blue cells contain a formula
- Note blank cells to take care of offset in ynu table



	A	B	C	D	E	F
1	f			100		
2	-ϕ			-0.01		
3	t		200		200	
4	y	0		20		0
5	nu		0.1		-0.1	

What you actually type into a formula cell is different from what it displays

Examples: Cell D2 contains the formula, = -1/D1

Cell D4 contains the formula, =C5*C3+B4

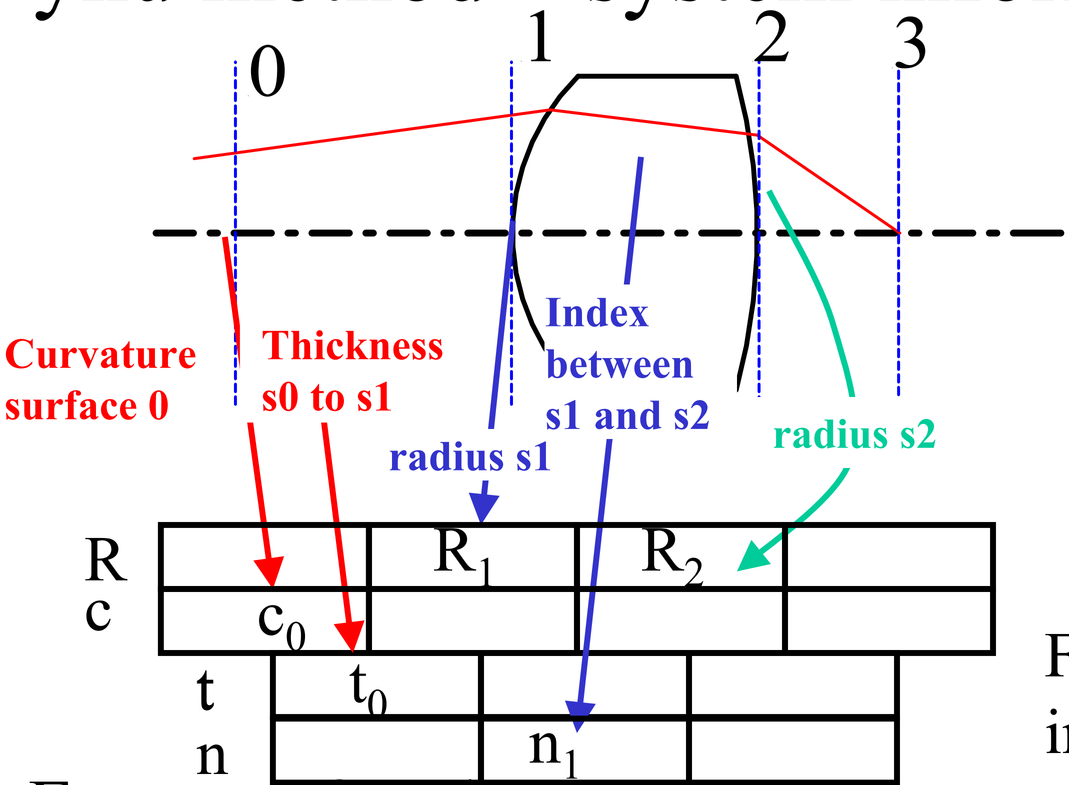
If you merge cells you can even get a staggered appearance

ynu method for refracting surfaces

Ynu raytracing can be used for single refracting surfaces, thick lenses, and even mirrors. The procedure is hardly different from what has already been done.

- For surfaces, each surface has a power
- Thicknesses between surfaces may be filled with a medium with an index>1

ynu method – system information at surfaces



Curvature surface 0

Thickness s0 to s1

radius s1

Index between s1 and s2

radius s2

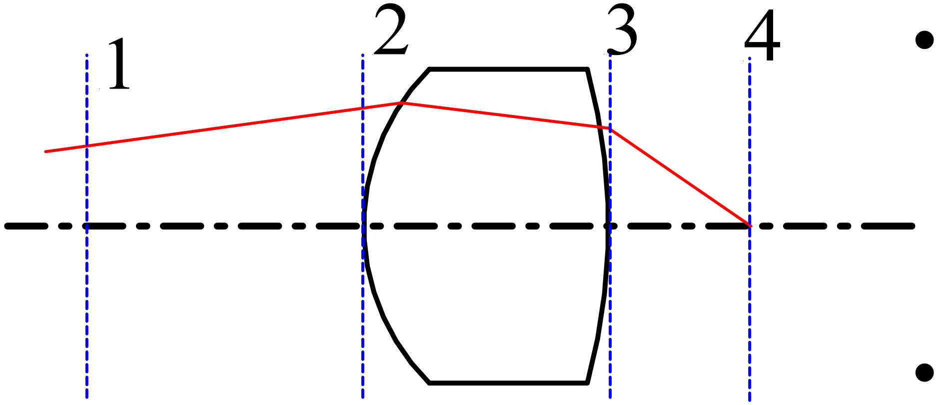
Curvature=1/radius

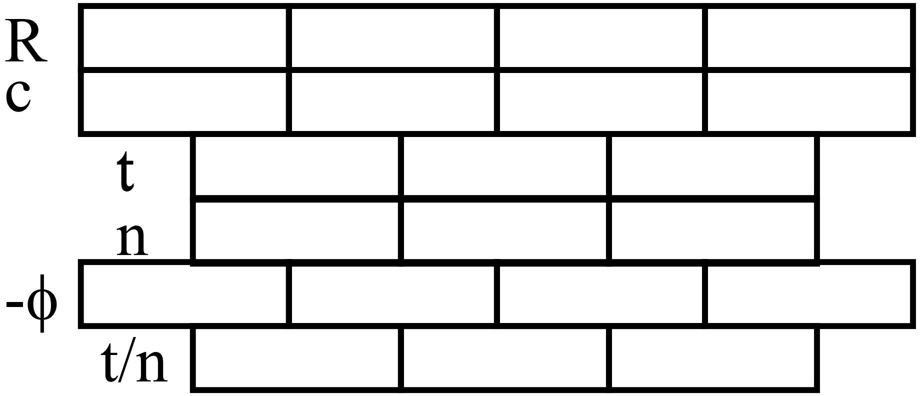
Important: the boxes are lined up staggered because R and c are properties of the surfaces; t and n are properties of the material between surfaces

For clarity not all data is indicated

- Every surface has a curvature
  - Plane surfaces (object, image, dummy) have zero curvature (just leave radius box blank)
- Every space between surfaces has a thickness and index
  - Numbers on thickness and index go with preceding surface
  - Some thicknesses (eg image distance) are not yet known

ynu method – refracting power, reduced thickness

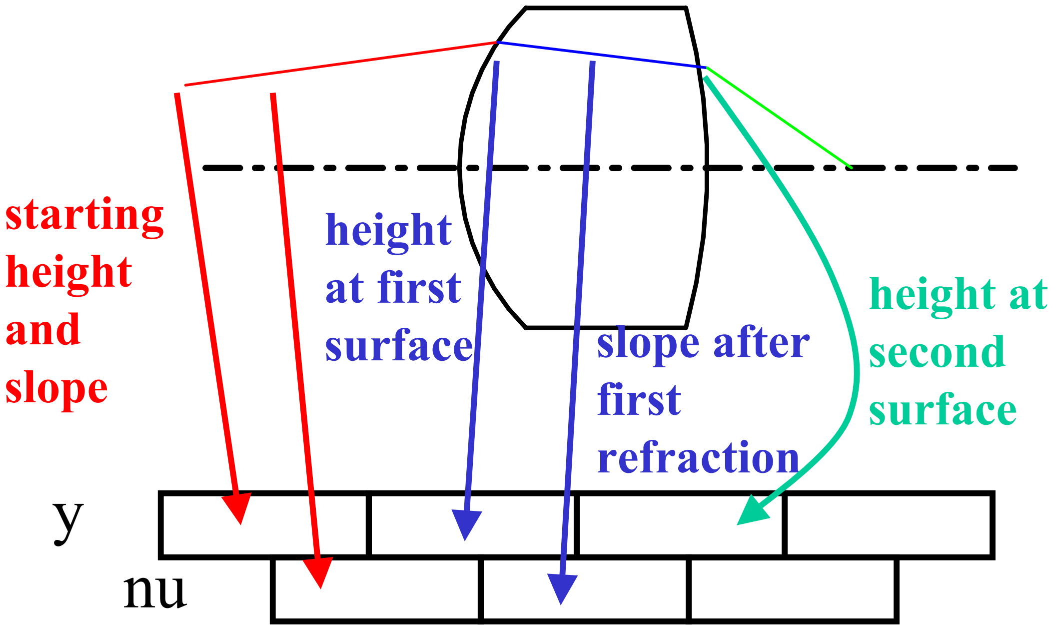




Note that boxes for negative power line up under surfaces, those for reduced thicknesses line up under spaces

- Every surface has a refracting power
  - How to calculate it will be given later
- Every space between surfaces has a "reduced" thickness
  - Equal to actual thickness divided by index
- Power and reduced thickness are the only system information needed for the raytrace

ynu raytracing-ray information



starting height and slope

height at first surface

slope after first refraction

height at second surface

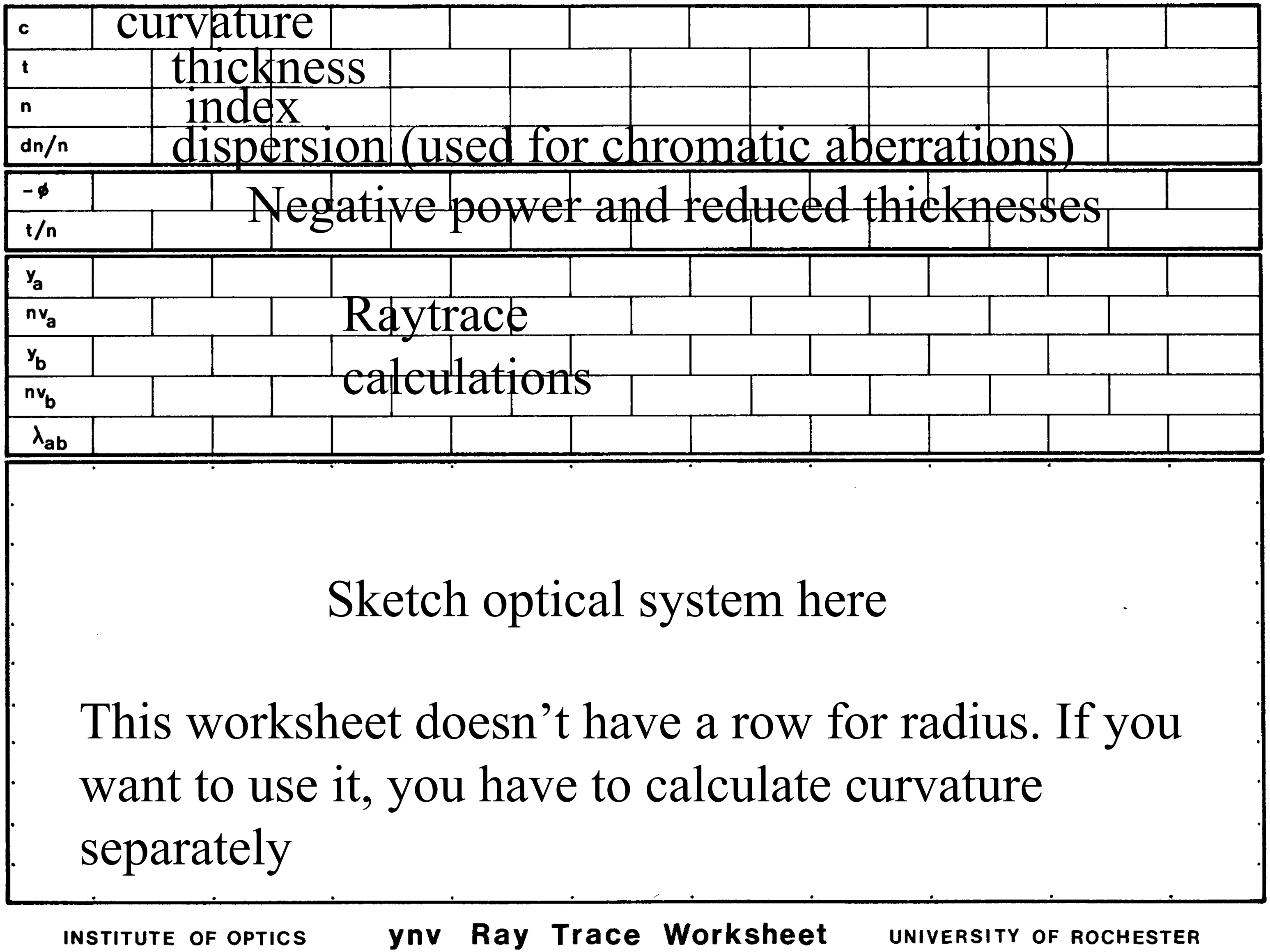
y

nu

With the exception of the starting height and slope, we don’t know any of these numbers yet. Finding them is the goal. This is just intended to show what is going to go in the boxes

- nu rather than u is used because it makes calculations easier
  - For thin lens in air, n=1 so it was left off
- Boxes are arranged in the order in which we calculate them from left to right
- “y” boxes will go under surface (curvature) boxes, “nu” boxes will go under space (thickness) boxes

ynu raytrace worksheet



c	curvature
t	thickness
n	index
dn/n	dispersion (used for chromatic aberrations)
$-\phi$	Negative power and reduced thicknesses
t/n	
y_a	Raytrace calculations
nv_a	
y_b	
nv_b	
λ_{ab}	

Sketch optical system here

This worksheet doesn’t have a row for radius. If you want to use it, you have to calculate curvature separately

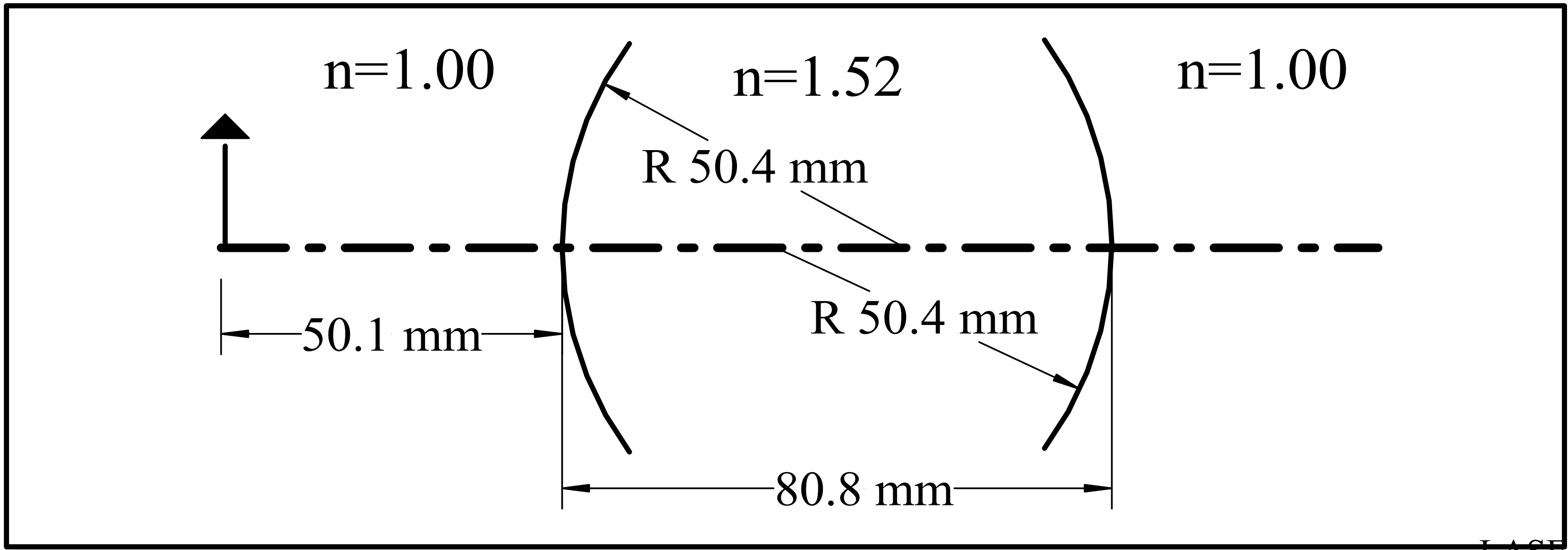
INSTITUTE OF OPTICS **ynv Ray Trace Worksheet** UNIVERSITY OF ROCHESTER

Entering data from sketch

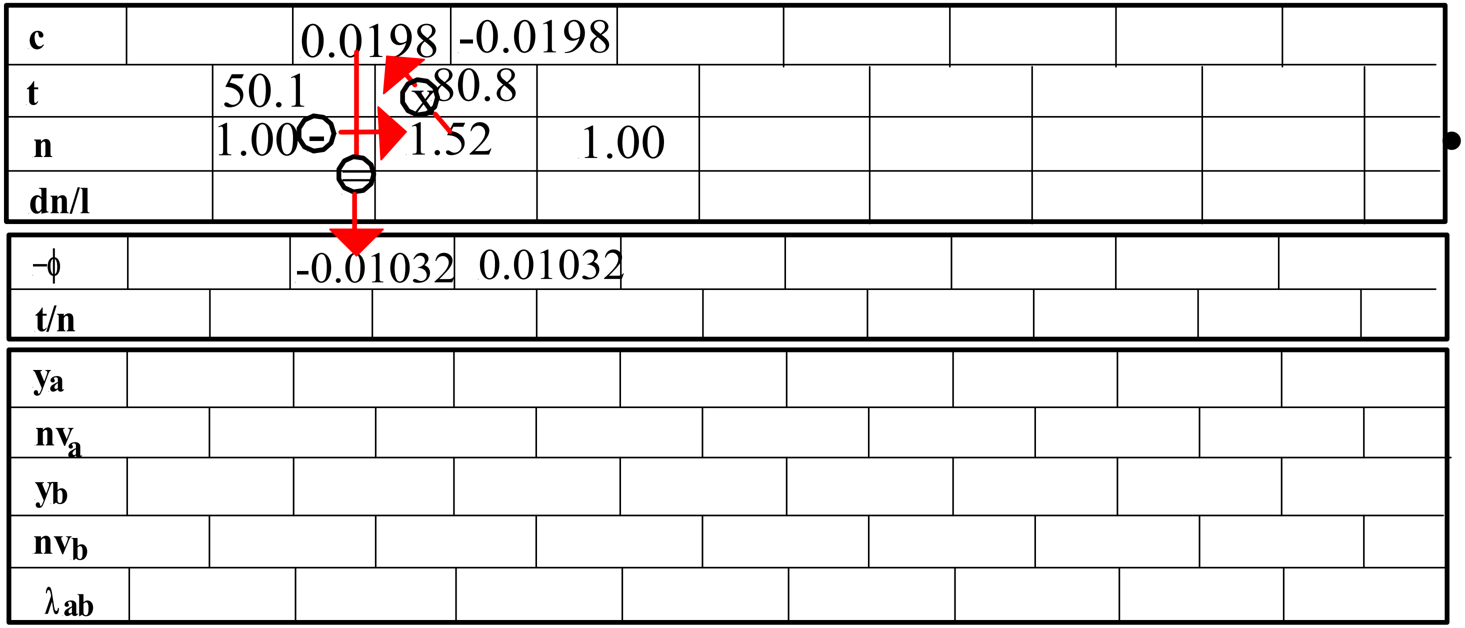
c		0.0198	-0.0198					
t	50.1	80.8						
n	1.00	1.52	1.00					
dn/l								

$-\phi$								
t/n								

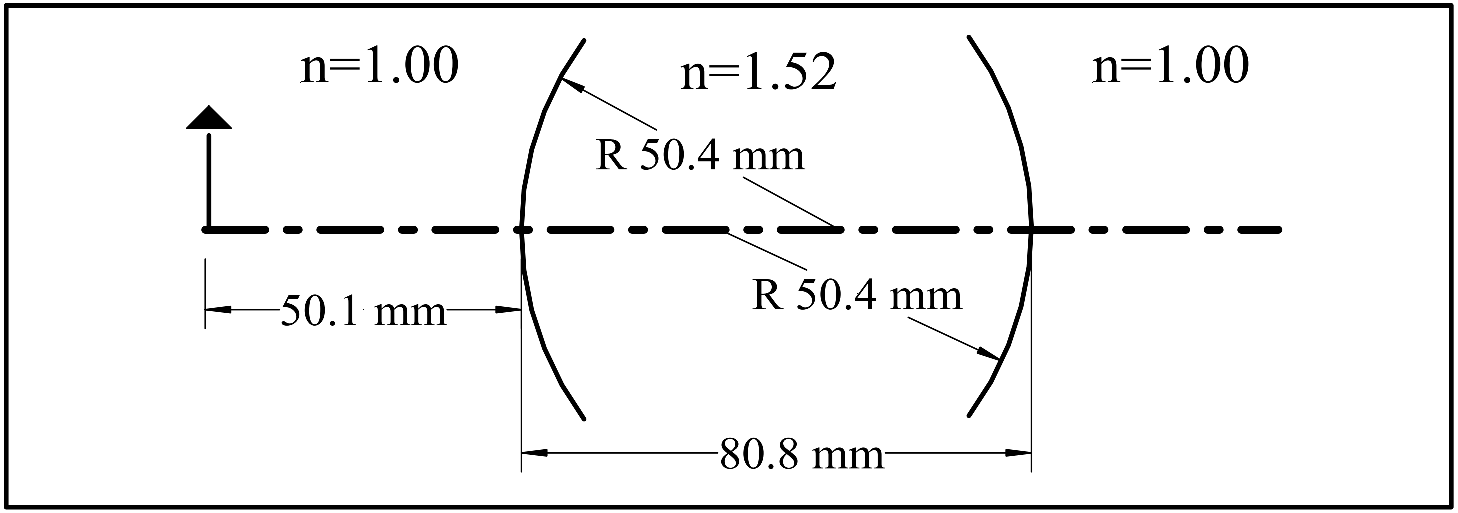
y_a								
nv_a								
y_b								
nv_b								
λ_{ab}								



Calculating negative power



c			0.0198		-0.0198	
t		50.1		-80.8		
n		1.00		1.52		1.00
dn/l						
−ϕ			-0.01032		0.01032	
t/n						
ya						
nv_a						
yb						
nv_b						
λ_{ab}						



- Keystrokes on TI-30 calculator

1.00

-

1.52

=

X

.01984

=

Power of surface and reduced thickness

Power of surface $\mathrm{i}, \phi_i = (n_i - n_{i-1}) c_i$

reduced thickness, $\tau_i = \dfrac{t_i}{n_i}$

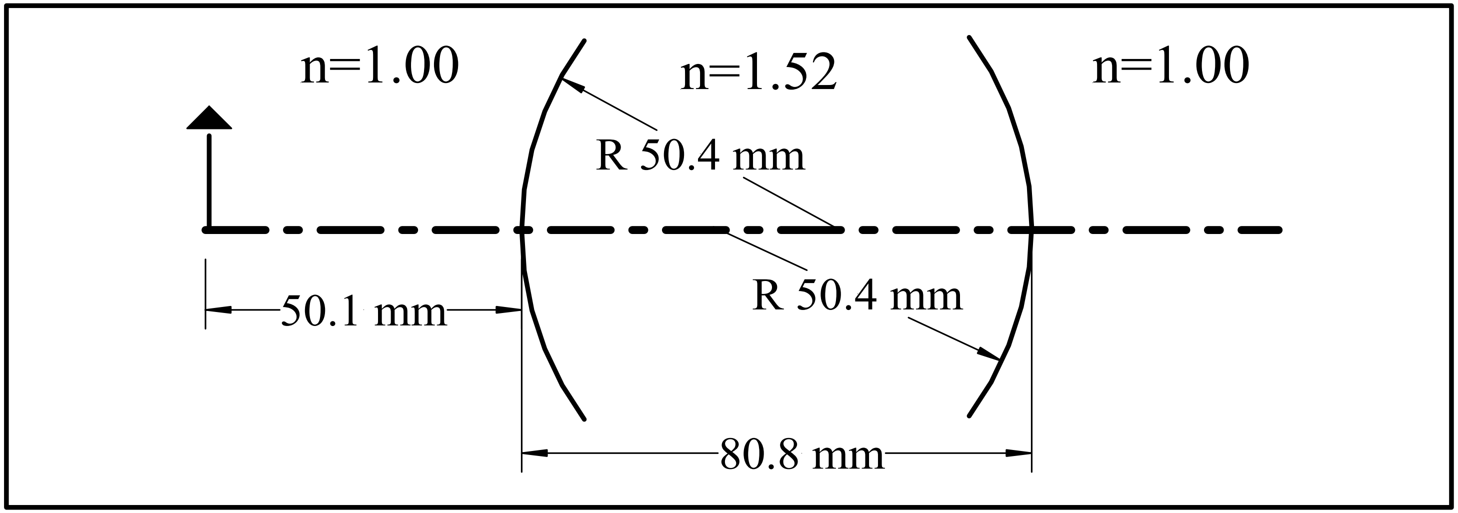
- Power of surface is like power of a thin lens but for only one surface
- These parameters are defined because they are used in the ynu raytracing formulae

Calculating reduced thickness

c		0.0198	-0.0198					
t	50.1	80.8						
n	1.00	1.52	1.00					
dn/l								

$-\phi$								
t/n	50.1	53.14						

y_a								
nv_a								
y_b								
nv_b								
λ_{ab}								



- Keystrokes on TI-30 calculator

50.10

÷

1.00

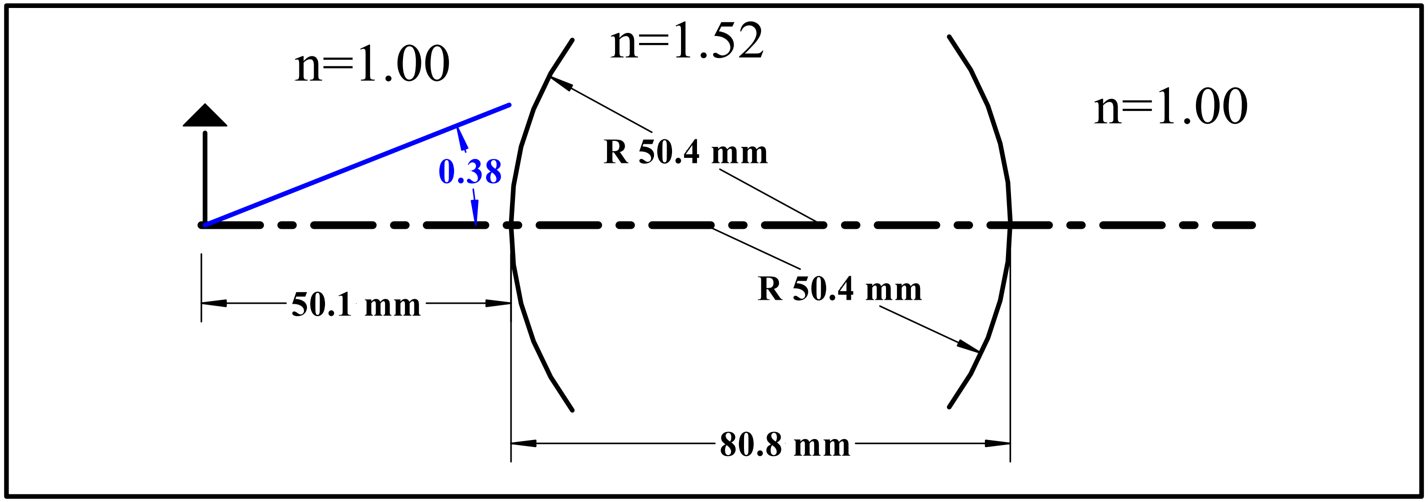
=

Beginning the raytrace-translation

c		0.0198	-0.0198					
t	50.1	80.8						
n	1.00	1.52	1.00					
dn/n								

$-\phi$		**-0.0103**	**-0.0103**					
t/n	**50.1**	**53.1**						

ya	**0**	**19.04**						
nv_a	**.38**							
yb								
nv_b								
λ ab								



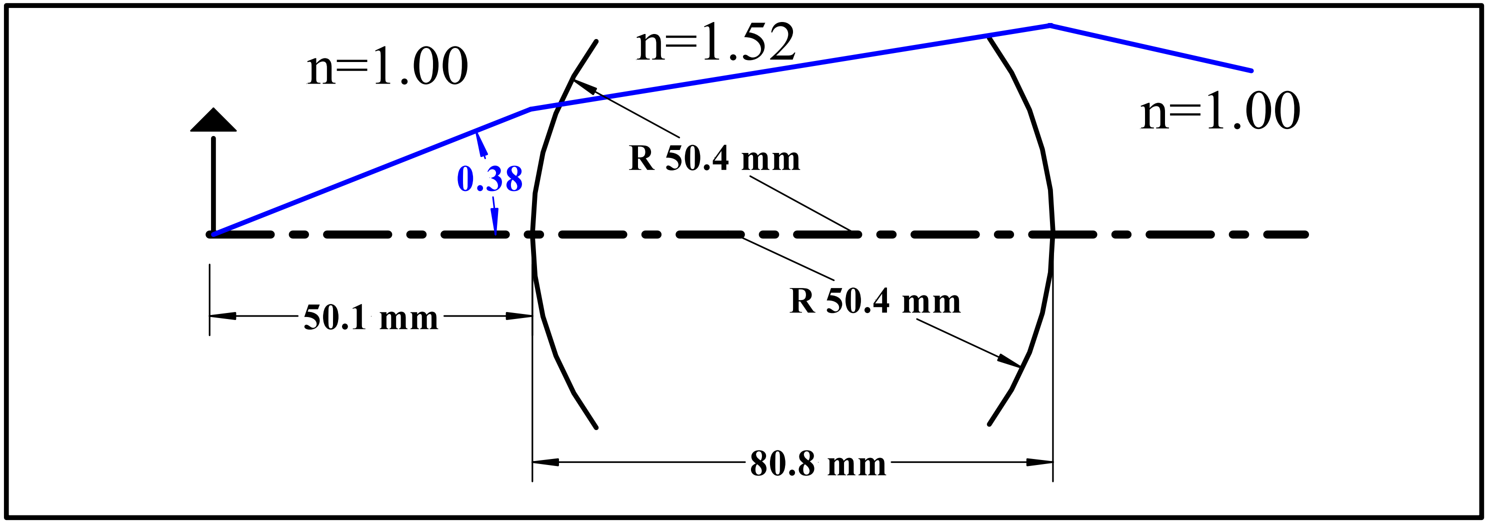
- Angles are all actually tangents (slopes)
- Keystrokes on TI-30

.38
X
50.1
=
+
0
=

Once the power and reduced thickness are found, the procedure is exactly as for thin lenses

Continuing the raytrace-refraction

c			0.0198		-0.0198											
t		50.1		80.8												
n		1.00		1.52		1.00										
dn/n																
$-\phi$			**-0.0103**		**-0.0103**											
t/n		**50.1**		**53.1**												
ya	**0**		**19.04**		**28.79**											
nv_a		**.38**		**0.184**		**-0.1136**										
yb																
nv_b																
λab																



- Keystrokes on TI-30 calculator

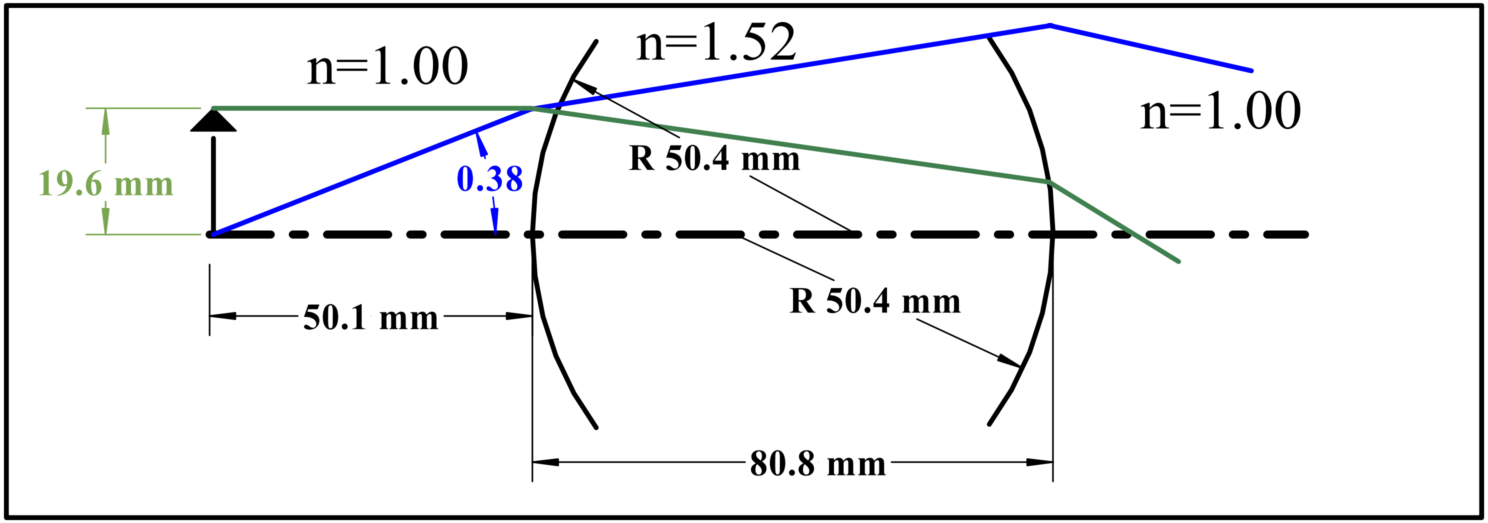
19.038
X
.01032
+ -
=
+
.38
=

Tracing another ray

c		0.0198	-0.0198					
t	50.1	80.8						
n	1.00	1.52	1.00					
dn/n								

$-\phi$		**-0.0103**	**-0.0103**					
t/n	**50.1**	**53.1**						

ya	**0**	**19.04**	**28.79**					
nv_a	**.38**	**0.184**	**-0.1136**					
yb	**19.6**	**19.6**	**8.86**					
nv_b	**0**	**-0.202**	**-0.294**					
λ_{ab}								

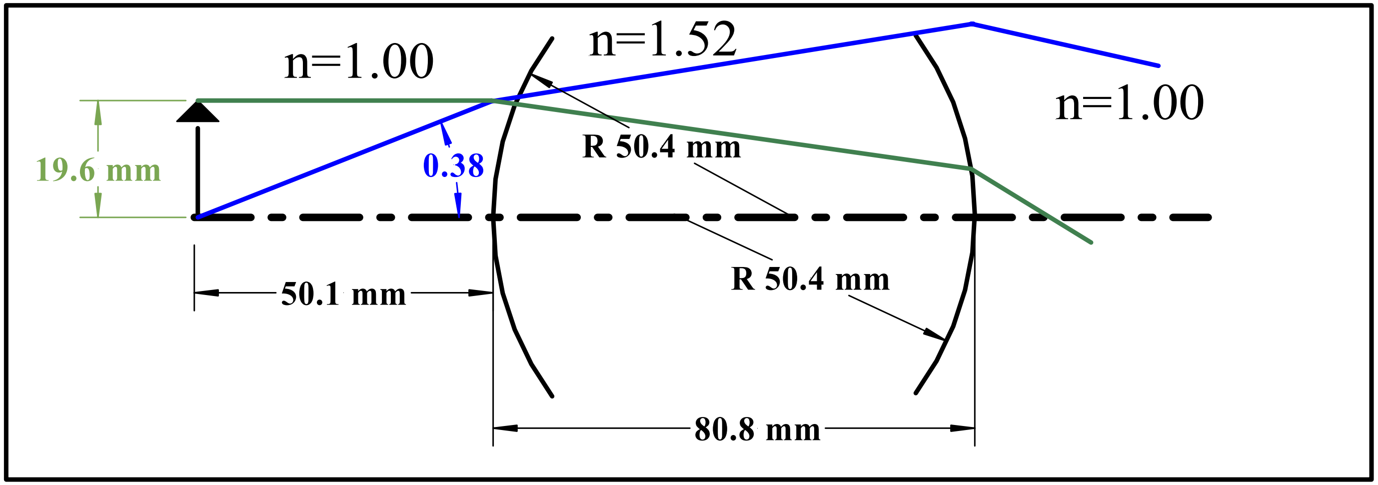


This works for any ray. It doesn’t have to start at the base or the tip of the object, or even be on the object.

Finding the image-height solves

c			0.0198		-0.0198		
t		50.1		80.8			
n		1.00		1.52		1.00	
dn/n							
$-\phi$			**-0.0103**		**-0.0103**		
t/n		**50.1**		**53.1**		**253.4**	
y_a	**0**		**19.04**		**28.79**		**0**
nv_a		**.38**		**0.184**		**-0.1136**	
y_b	**19.6**		**19.6**		**8.86**		**-65.61**
nv_b		**0**		**-0.202**		**-0.294**	
λ_{ab}							

Desired height (image location)



- Keystrokes on TI-30 calculator

0

-

28.791

=

÷

.1136

+ ⟲ -

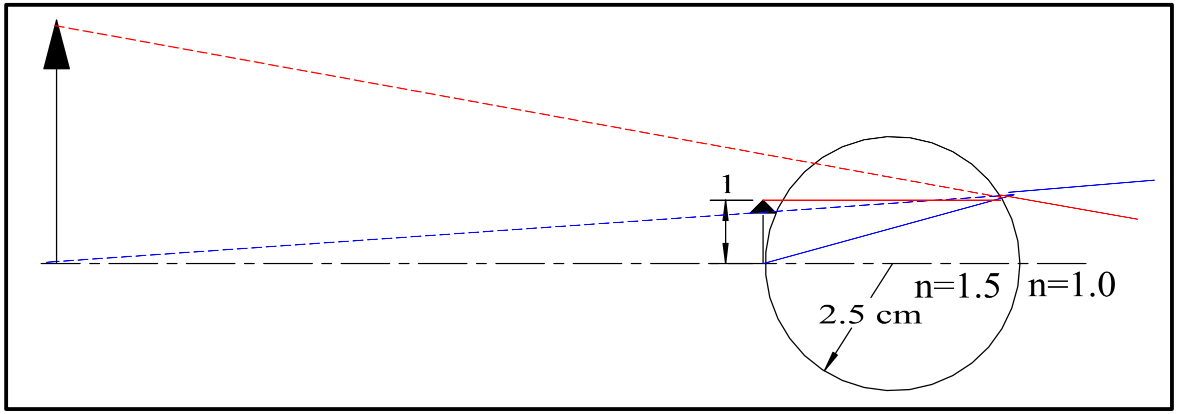
=

Desired height doesn’t have to be 0. It was zero here because we wanted an image.

A simple application

4. A sphere 5 cm in diameter has a small scratch on its surface. When the scratch is viewed *through* the glass from a position directly opposite, where does the scratch appear? What is its magnification? Assume n = 1.50 for the glass.

c			-0.4		
t		5.			
n		1.5		1.0	
dn/n					
$-\phi$			-0.2		
t/n		3.333		-10	
y_a	0		3.333		0
nv_a		1		0.333	
y_b	1		1		6.0
nv_b		0		-0.2	
λ_{ab}					



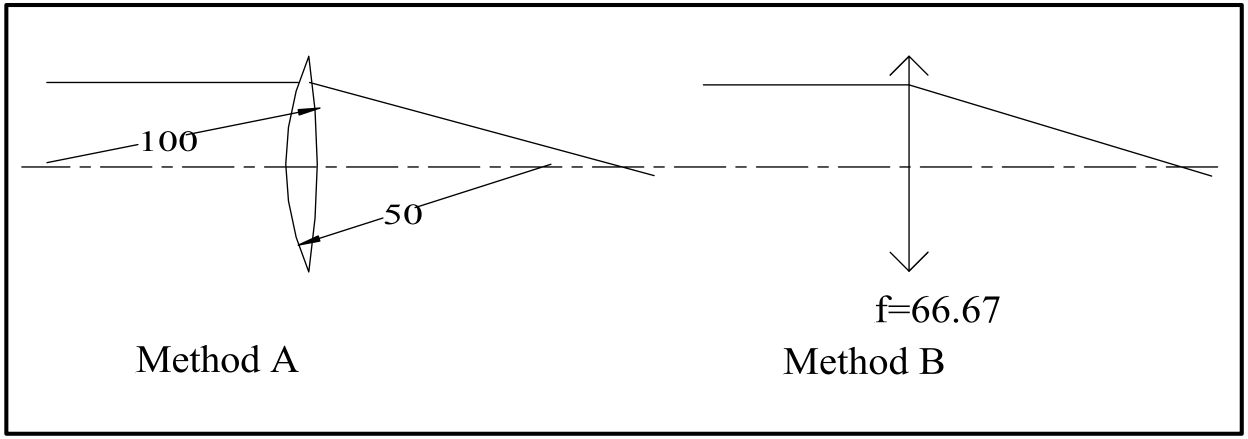
Note that the slope of the a-ray doesn't matter since all we care about is where the height is zero. Similarly, the height of the b-ray doesn't matter.

- Black=input numbers
- Magenta=power and reduced thickness calculation
- Blue, red = raytrace
- Green=final answer

Ynu raytracing with thin lenses-two ways

- Method A
  - given surface radii, index
  - enter zero thickness for lens
  - focal length found by thickness solve
- Method B
  - given focal length
  - leave surface data blank
  - enter negative surface power in -ϕ row

c			-0.01		0.02														
t		∞		0.															
n		1.0		1.5		1.0													
dn/n																			
$-\phi$			-0.005		-0.01										-0.015				
t/n		∞		0.		66.67								∞		66.67			
y_a	1		1		1		0						1		1		0		
nv_a		0		-0.005		-0.015								0		-0.015			
y_b																			
nv_b																			
λ_{ab}																			

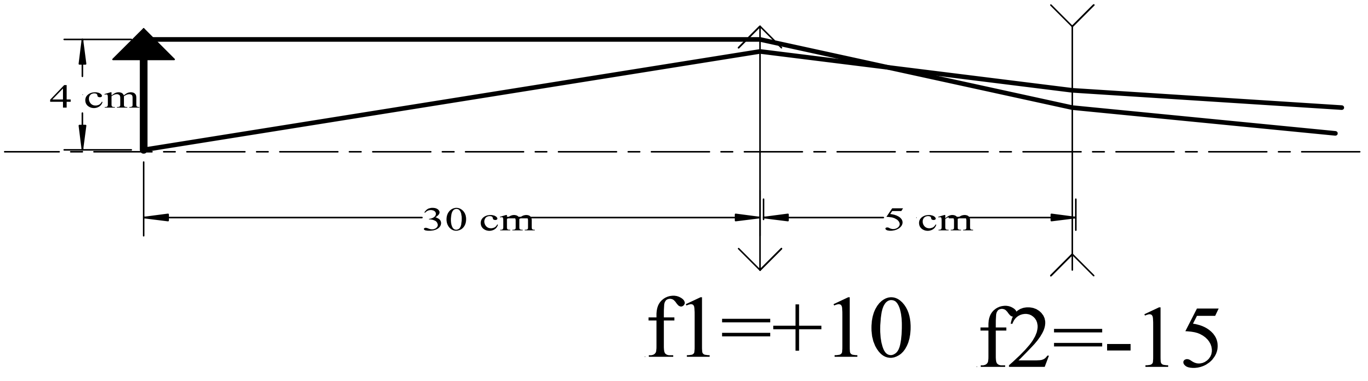


A thin-lens example

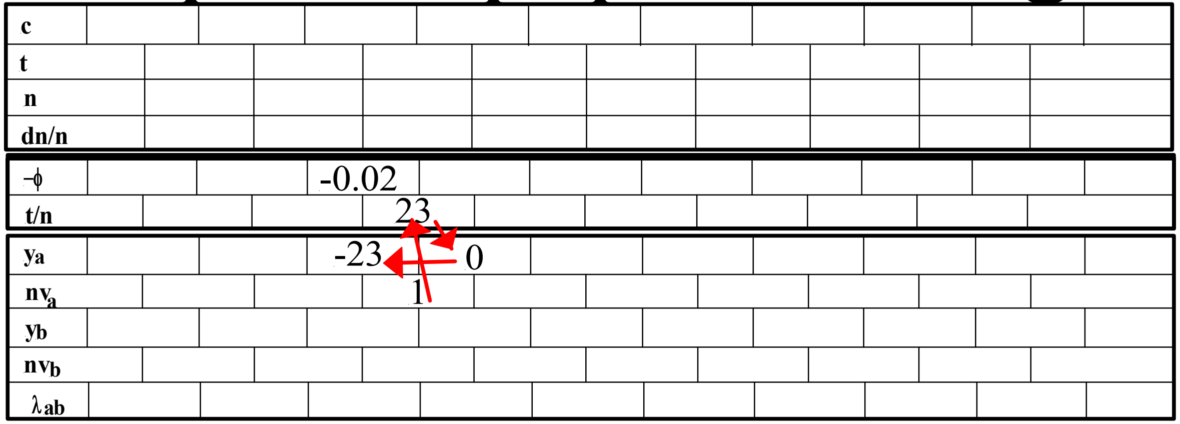
1. **Two thin lenses with focal lengths $f_1 = +10$ cm and $f_2 = -15$ cm are placed 5 cm apart. If an object 4 cm high is located 30 cm to the left of the first lens, find the following:**
   a. **Position, size and character of the final image.**
   b. **Lateral magnification, using both mathematical and graphical ray tracing methods.**

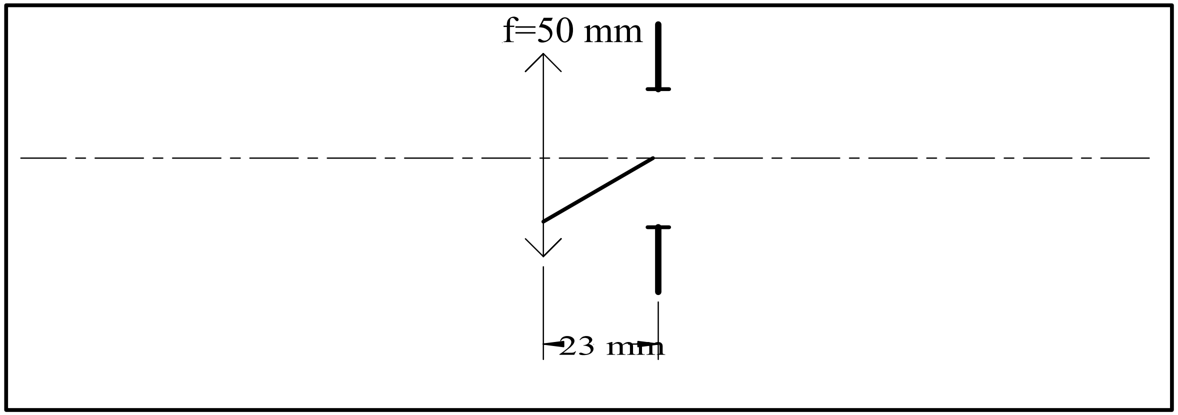
c										
t										
n										
dn/n										
$-\phi$		-0.1	0.0667							
t/n	30	5	30							
y_a	0	30	20	0						
nv_a	1	-2	-0.6667							
y_b	4	4	2	-6						
nv_b	0	-0.4	-0.2667							
λ_{ab}										

Thickness solve



Stops and pupils-tracing rays backwards





- Translation
- Keystrokes for TI-30

1

X

23

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+ ⟳ -

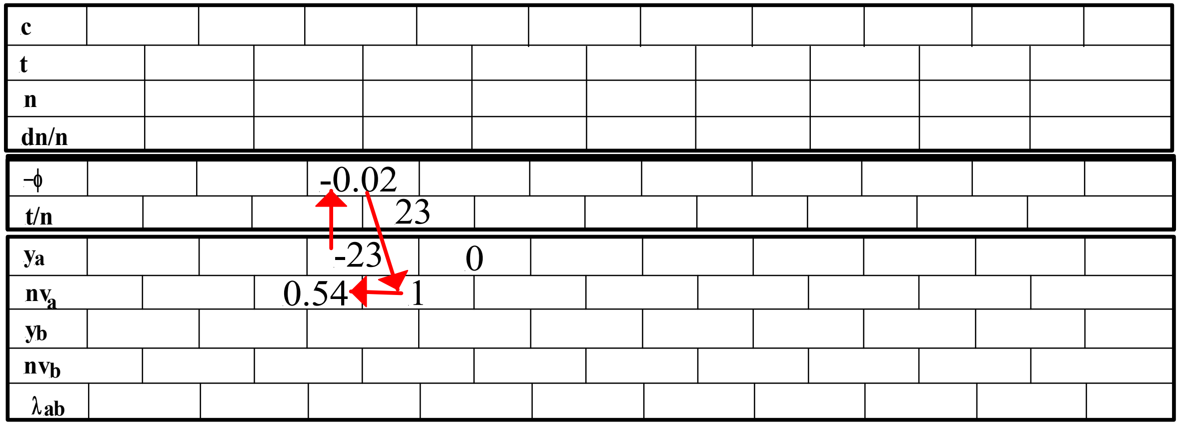
+

0

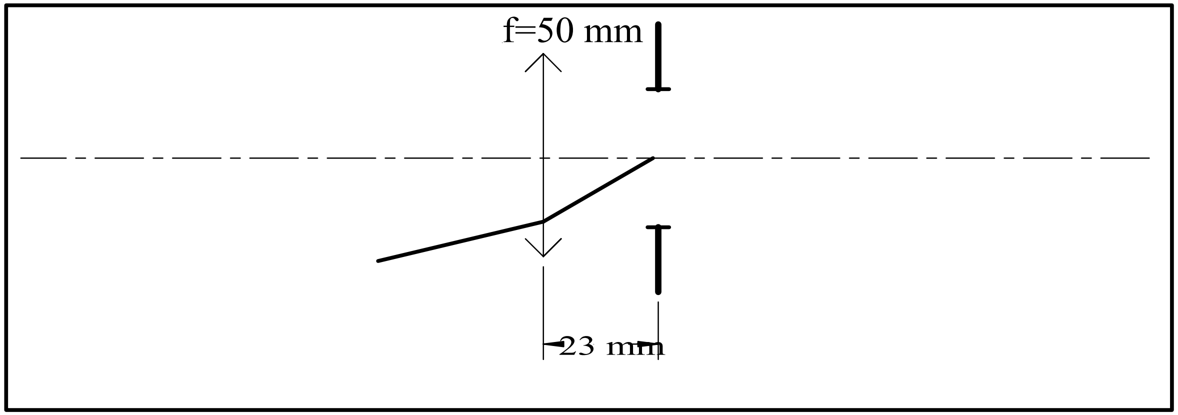
=

- To find chief ray and location of aperture find ray which goes through center of stop
- To trace rays backwards, use minus instead of plus on product

Tracing rays backwards-refraction



c																			
t																			
n																			
dn/n																			
$-\phi$					-0.02														
t/n						23													
y_a					-23		0												
nv_a				0.54		1													
y_b																			
nv_b																			
λ_{ab}																			



- Keystrokes for TI-30 calculator

23

+⟳-

X

.02

+⟳-

=

+⟳-

+

1

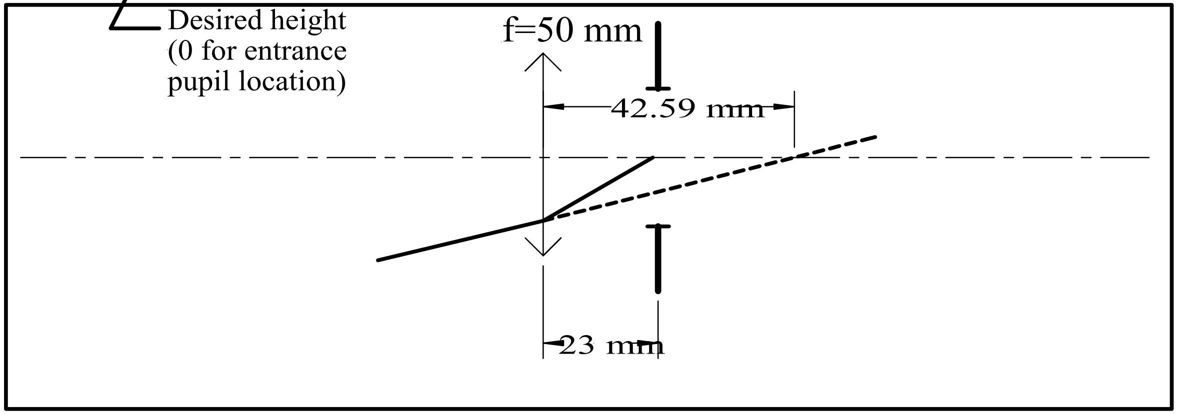
=

Again just like forward refraction except the product comes in with a minus sign.

Backwards raytracing-thickness solve, entrance pupil

c								
t								
n								
dn/n								
-ϕ					-0.02			
t/n				-42.59		23		
ya			0		-23		0	
nv$_a$				0.54		1		
yb								
nv$_b$								
λab								

Desired height (0 for entrance pupil location)



Why is distance negative?

Sign convention, thickness is positive if higher number surface is to right of lower numbered surface

- Like forward solve except desired height on left
- Keystrokes for TI30

23

+⇄ -

-

0

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.54

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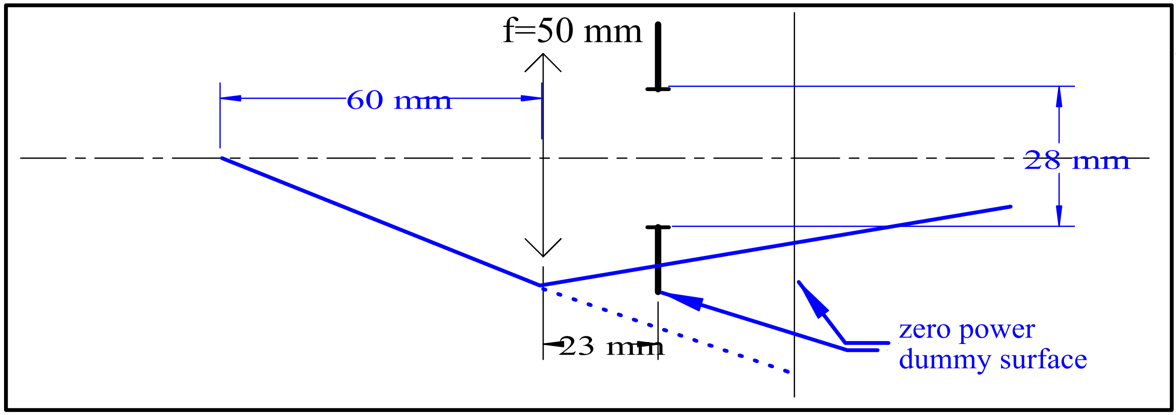
Size of entrance pupil-dummy surfaces, scaling rays

c									
t									
n			Entrance				Aperture		
dn/n			pupil		lens		stop		
-ϕ			0		-0.02		0		
t/n		102.59		-42.59		23		277	
ya			0		-23		0		
nva				0.54		1			
yb	0		102.59		60		55.4		0
nvb		1		1		-0.2		-0.2	
λab									

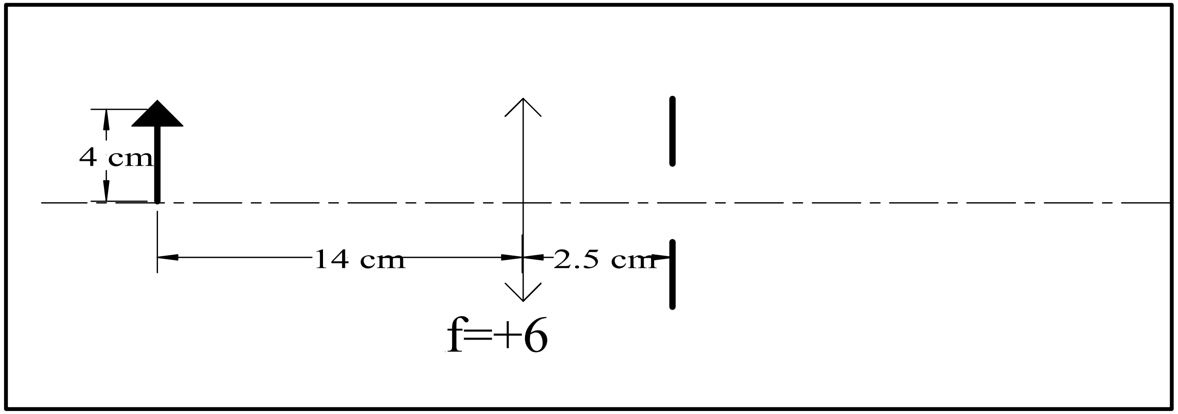


- A zero-power surface can be inserted anywhere
  - useful for stops, image shifts, etc.
  - calculation is unchanged
- Any ray can be multiplied by any number to get a new ray
  - here we multiply by 14/55.4=0.253 to get a ray which just hits aperture stop
  - ray height at entrance pupil is then 0.253*102.59=25.9

Aperture stop, entrance pupil example

3. **Repeat Exercise 2 for an object 4 cm high, with a 2-cm aperture stop and a thin convex lens of 6-cm focal length and 5-cm aperture. The object is 14 cm ~~high~~ in front of the lens. The stop is 2.50 cm behind the lens.**

c										
t										
n										
dn/n										
−φ			-0.1667							
t/n		14	10.5							
y_a		0	14	0						
nv_a		1	-0.1333							
y_b		4	4	-3						
nv_b		0	-0.6667							
λ_{ab}										

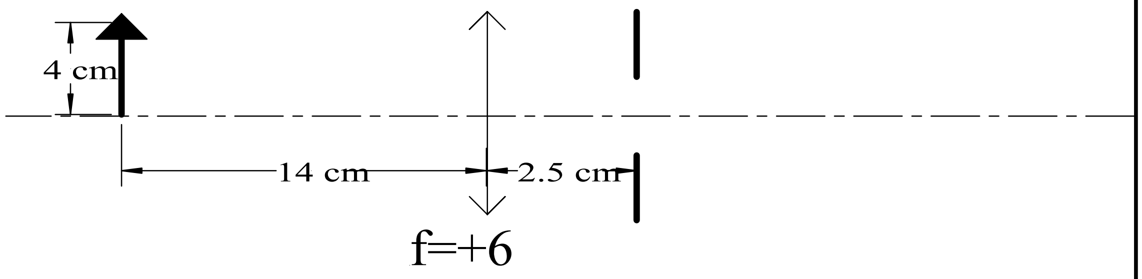


- Size and location of image

Aperture stop, entrance pupil example

3. **Repeat Exercise 2 for an object 4 cm high, with a 2-cm aperture stop and a thin convex lens of 6-cm focal length and 5-cm aperture. The object is 14 ~~cm high~~ in front of the lens. The stop is 2.50 cm behind the lens.**

c																			
t																			
n																			
dn/n																			
$-\phi$					-0.1667		0												
t/n				-4.286		2.5													
y_a			0		-2.5		0												
nv_a				0.583		1													
y_b			1.714		1		1												
nv_b				0.1667		0													
λ_{ab}																			



- Size and location of entrance pupil