Loop Gain

Contrary to amplifying the radiation, there are many **losses**:

- ➢ Scattering and absorption losses at the end mirrors.
- ➢ Output radiation through the output coupler.
- ➢ Scattering and absorption losses in the active medium, and at the side walls of the laser.
- ➢ Diffraction losses because of the finite size of the laser components.

These losses cause some of the radiation not to take part the lasing process.

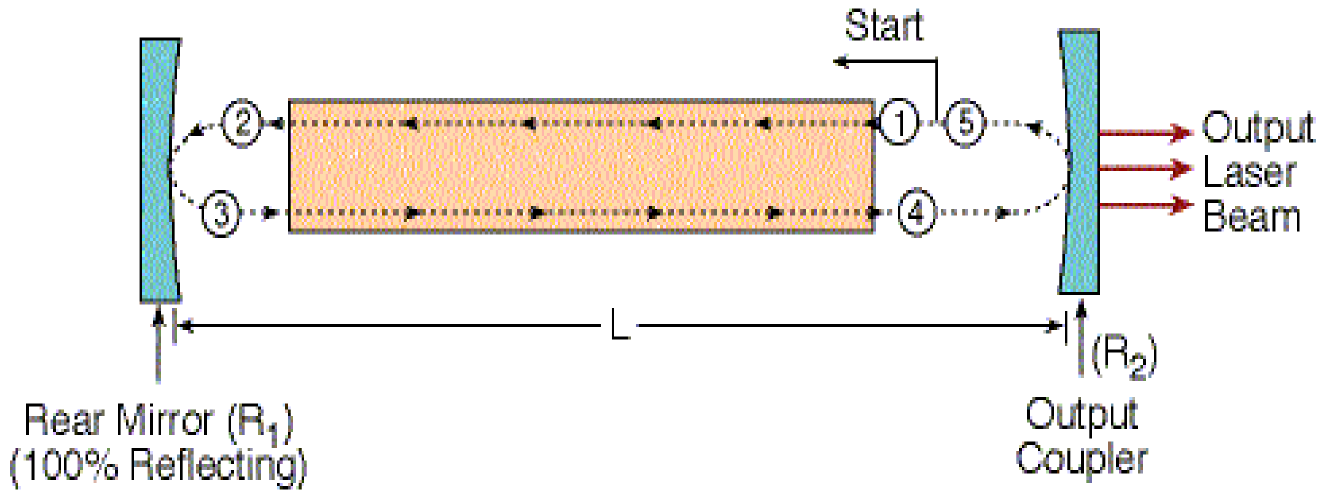
A necessary condition for lasing is that the total gain will be a little higher than all the losses.

Loop Gain (G_L)

The round trip path of the radiation through the laser cavity:
The path is divided to sections numbered by 1-5, while point
same point as "1". By definition, **Loop Gain** is given by:

$$G_L = E_5/E_1$$

G_L = Loop Gain.
E $=_1$ Intensity of radiation at the beginning of the loop.
E_5 = Intensity of radiation at the end of the loop.



Calculating Loop Gain (G_L) Without Losses

G_A = Active medium gain

$E_2 = G_A * E_1$

length of the cavity, such that the active medium feel the length of the laser cavity.

On the way from point “2” to point “3”, As a result: **$E_3 = R_1 * G_A * E_1$**

through the active medium, and amplified. Thus: **$E_4 = R_1 * G_A^2 * E_1$**

the output coupler, which have a reflectivity R_2. Thus: **$E_5 = R_1 * R_2 * G_A^2 * E_1$**

This completes the loop.

Calculating Loop Gain (G_L) With Losses

We **assume that the losses occur uniformly along the length cavity (L)**.

In analogy to the Lambert formula for losses , we define **loss coefficient (a)**, **absorption factor M**: **M = exp(-2aL)**

a= **Loss coefficient**

2L = **Path Length**, which is twice the length of the cavity.

Adding the **loss factor** **(M)** to the equation of E_5:

$$E_5 = R_1 * R_2 * G_A^2 * E_1 * M$$

From this we can calculate the **Loop gain**:

$$G_L = E_5 / E_1 = R_1 * R_2 * G_A^2 * M$$

Calculating Gain Threshold $(G_L)_{th}$

As we assumed uniform distribution of the **loss coefficient (a)**,
we now define **gain coefficient (b)**, and assume **active medium gain (G_A)** as distributed uniformly along the length of the cavity.

$$G_A = \exp(+bL)$$

Substituting the last equation in the Loop Gain:

$$G_L = R_1 * R_2 * \exp(2(b-a)L))$$

Conclusion:

There is a threshold condition for amplification, in order to create oscillation inside the laser. This Threshold Gain is marked with index **"th"**.

For continuous laser , the threshold condition is:

$$(G_L)_{th} = 1 = R_1 R_2 G_A M = R_1 * R_2 * \exp(2(b-a)L)$$

Mathematical Expressions of fluorescence linewidth

Fluorescence linewidth is expressed by wavelengths, or frequencies, of two points on the spontaneous emission graph at **half the maximum height**.

$$\Delta\nu = |\nu_2 - \nu_1| = \left|\frac{c}{\lambda_2} - \frac{c}{\lambda_1}\right| = \left|\frac{c\lambda_1 - c\lambda_2}{\lambda_1\lambda_2}\right| = \frac{c\Delta\lambda}{\lambda_1\lambda_2}$$

The linewidth (D l) is much smaller than each of the wavelengths ($Dl << l_1, l_2$).

Thus the **approximation:** $l_1 * l_2 =$ can be used.

l_0 = Wavelength at the center of emission spectrum of the laser.

The result is:

In a similar way:

these mathematical relations will be used for determining the **coherence** of the laser.

Broadening the Fluorescence line

Certain mechanisms are responsible for broadening the linewidth of a laser:

1. **Natural broadening.**
2. **Doppler Broadening.**
3. **Pressure broadening.**

For many applications, especially when **temporal coherence** is required a **small linewidth of the emitted laser wavelength is required**.

Natural broadening.

This broadening is always present, and comes from the **finite transition time from the upper laser level to the lower laser level.**

Natural linewidth is narrow: 10^4 - 10^8 [Hz], compared to the radiation frequency of visible light: 10^{14} [Hz].

Each energy level has a specific width (Dn), and specific lifetime (Dt).
Natural broadening results from the **Heisenberg uncertainty principle:**

D**E*** D**t > h**

D**E = h***

Dn Dn > 1/

D**t**

Numerical examples:

Dt = 10^{-8} [s] = = > Dn = 10^8 [Hz]

Dt = 10^{-4} [s] = = > Dn = 10^4 [Hz]

The longer the specific energy level transition lifetime, the narrower is its linewidth.

Doppler Broadening

Doppler shift is a well known phenomena in wave motion.

It occurs when the source is in relative motion to the receiver.

The frequency detected is shifted by an amount determined by the **relative velocity between the source and the receiver.**

Since **gas molecules are in constant motion in random directions**, each molecule emit light while it is moving relative to the laser axis in a different direction. These distribution of frequency shifts cause the broadening of the laser linewidth.

Doppler broadening occur especially in **gas lasers**, as a result of movement of gas molecules.

Its influence is mostly in **low pressure gas lasers**

Pressure (collisions) broadening

It is caused by collisions between the molecules of the gas.
Pressure broadening is the **largest broadening mechanism in gas lasers** with pressure of more than 10 mm Hg.
As the pressure increase, the broadening increase.

At constant pressure (P), as the temperature (T) increases:

PV = nRT
P = const = = > V increases when T increases.
Since the Volume (V) increases, the number of collisions decrease. Thus, pressure ((collisions) broadening decrease.

Numerical example:

1. At room temperature, the linewidth of CO_2 laser with gas pressure of 10 [torr] is 55 [MHz].
2. At room temperature, the linewidth of CO_2 laser with gas pressure of 100 [torr] is 500 [MHz].
3. Above 100 [torr], the increase rate of broadening is about 6.5 [MHz] for each increase in pressure of 1 [torr].

Linewidth broadening

the result of broadening of the fluorescence linewidth.