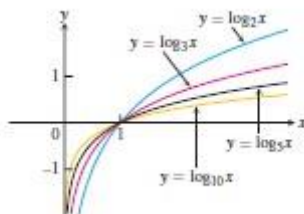


: logarithmic function 4.8

These are the functions where the base is a positive constant. They are the *inverse functions* of the exponential functions, Figure below shows the graphs of four logarithmic functions with various bases



: Transcendental Functions 4.9

These are functions that are not algebraic. They include the trigonometric, inverse trigonometric, exponential, and logarithmic functions, and many other functions as well. A particular example of a transcendental function is a **catenary**. Its graph has the shape of a cable, like a telephone line or electric cable, strung from one support to another and hanging freely under its own weight

Functions

In Exercises 1–6, find the domain and range of each function.

- $f(x) = 1 + x^2$
- $f(x) = 1 - \sqrt{x}$
- $f(x) = \sqrt{5x + 10}$
- $g(x) = \sqrt{x^2 - 3x}$
- $f(t) = \frac{4}{3 - t}$
- $G(t) = \frac{2}{t^2 - 16}$

Functions and Graphs

Find the domain and graph the functions in Exercises 15–20.

- $f(x) = 5 - 2x$
- $f(x) = 1 - 2x - x^2$
- $g(x) = \sqrt{|x|}$
- $g(x) = \sqrt{-x}$
- $F(t) = t/|t|$
- $G(t) = 1/|t|$
- Find the domain of $y = \frac{x + 3}{4 - \sqrt{x^2 - 9}}$
- Find the range of $y = 2 + \frac{x^2}{x^2 + 4}$

Piecewise-Defined Functions

Graph the functions in Exercises 25–28.

- $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - x, & 1 < x \leq 2 \end{cases}$
- $g(x) = \begin{cases} 1 - x, & 0 \leq x \leq 1 \\ 2 - x, & 1 < x \leq 2 \end{cases}$
- $F(x) = \begin{cases} 4 - x^2, & x \leq 1 \\ x^2 + 2x, & x > 1 \end{cases}$

Increasing and Decreasing Functions

Graph the functions in Exercises 37–46. What symmetries, if any, do the graphs have? Specify the intervals over which the function is increasing and the intervals where it is decreasing.

- $y = -x^3$
- $y = -\frac{1}{x^2}$
- $y = -\frac{1}{x}$
- $y = \frac{1}{|x|}$
- $y = \sqrt{|x|}$
- $y = \sqrt{-x}$
- $y = x^3/8$
- $y = -4\sqrt{x}$
- $y = -x^{3/2}$
- $y = (-x)^{2/3}$

Even and Odd Functions

In Exercises 47–58, say whether the function is even, odd, or neither. Give reasons for your answer.

- $f(x) = 3$
- $f(x) = x^5$
- $f(x) = x^2 + 1$
- $f(x) = x^2 + x$
- $g(x) = x^3 + x$
- $g(x) = x^4 + 3x^2 - 1$
- $g(x) = \frac{1}{x^2 - 1}$
- $g(x) = \frac{x}{x^2 - 1}$
- $h(t) = \frac{1}{1 - t}$
- $h(t) = |t^3|$
- $h(t) = 2t + 1$
- $h(t) = 2|t| + 1$

Combining Functions; Shifting and Scaling Graphs .5

Sums, Differences, Products, and Quotients

Like numbers, functions can be added, subtracted, multiplied, and divided (except where the denominator is zero) to produce new functions. If f and g are functions, then for every x that belongs to the domains of both f and g we define

$$\begin{aligned}(f + g)(x) &= f(x) + g(x). \\ (f - g)(x) &= f(x) - g(x). \\ (fg)(x) &= f(x)g(x).\end{aligned}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad (\text{where } g(x) \neq 0).$$

EXAMPLE 1 The functions defined by the formulas

$$f(x) = \sqrt{x} \quad \text{and} \quad g(x) = \sqrt{1-x}$$

have domains $D(f) = [0, \infty)$ and $D(g) = (-\infty, 1]$. The points common to these domains are the points

$$[0, \infty) \cap (-\infty, 1] = [0, 1].$$

The following table summarizes the formulas and domains for the various algebraic combinations of the two functions. We also write $f \cdot g$ for the product function fg .

Function	Formula	Domain
$f + g$	$(f + g)(x) = \sqrt{x} + \sqrt{1-x}$	$[0, 1] = D(f) \cap D(g)$
$f - g$	$(f - g)(x) = \sqrt{x} - \sqrt{1-x}$	$[0, 1]$
$g - f$	$(g - f)(x) = \sqrt{1-x} - \sqrt{x}$	$[0, 1]$
$f \cdot g$	$(f \cdot g)(x) = f(x)g(x) = \sqrt{x(1-x)}$	$[0, 1]$
f/g	$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \sqrt{\frac{x}{1-x}}$	$[0, 1)$ ($x = 1$ excluded)
g/f	$\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \sqrt{\frac{1-x}{x}}$	$(0, 1]$ ($x = 0$ excluded)

Composite Functions

Composition is another method for combining functions.

DEFINITION If f and g are functions, the **composite function** $f \circ g$ ("f composed with g") is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f .

EXAMPLE 2 If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find

- (a) $(f \circ g)(x)$ (b) $(g \circ f)(x)$ (c) $(f \circ f)(x)$ (d) $(g \circ g)(x)$.

Solution

Composite	Domain
(a) $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{x+1}$	$[-1, \infty)$
(b) $(g \circ f)(x) = g(f(x)) = f(x) + 1 = \sqrt{x} + 1$	$[0, \infty)$
(c) $(f \circ f)(x) = f(f(x)) = \sqrt{f(x)} = \sqrt{\sqrt{x}} = x^{1/4}$	$[0, \infty)$
(d) $(g \circ g)(x) = g(g(x)) = g(x) + 1 = (x+1) + 1 = x+2$	$(-\infty, \infty)$

To see why the domain of $f \circ g$ is $[-1, \infty)$, notice that $g(x) = x + 1$ is defined for all real x but belongs to the domain of f only if $x + 1 \geq 0$, that is to say, when $x \geq -1$. ■

Notice that if $f(x) = x^2$ and $g(x) = \sqrt{x}$, then $(f \circ g)(x) = (\sqrt{x})^2 = x$. However, the domain of $f \circ g$ is $[0, \infty)$, not $(-\infty, \infty)$, since \sqrt{x} requires $x \geq 0$.

Shifting a Graph of a Function

A common way to obtain a new function from an existing one is by adding a constant to each output of the existing function, or to its input variable. The graph of the new function is the graph of the original function shifted vertically or horizontally, as follows.

Shift Formulas

Vertical Shifts

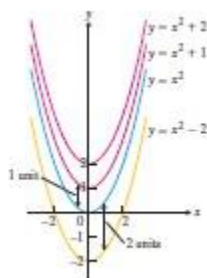
$y = f(x) + k$ Shifts the graph of f up k units if $k > 0$
Shifts it down $|k|$ units if $k < 0$

Horizontal Shifts

$y = f(x + h)$ Shifts the graph of f left h units if $h > 0$
Shifts it right $|h|$ units if $h < 0$

EXAMPLE 3

- (a) Adding 1 to the right-hand side of the formula $y = x^2$ to get $y = x^2 + 1$ shifts the graph up 1 unit (Figure 1.29).
- (b) Adding -2 to the right-hand side of the formula $y = x^2$ to get $y = x^2 - 2$ shifts the graph down 2 units (Figure 1.29).
- (c) Adding 3 to x in $y = x^2$ to get $y = (x + 3)^2$ shifts the graph 3 units to the left (Figure 1.30).
- (d) Adding -2 to x in $y = |x|$, and then adding -1 to the result, gives $y = |x - 2| - 1$ and shifts the graph 2 units to the right and 1 unit down (Figure 1.31). ■



Scaling and Reflecting a Graph of a Function

To scale the graph of a function $y = f(x)$ is to stretch or compress it, vertically or horizontally. This is accomplished by multiplying the function f , or the independent variable x , by an appropriate constant c . Reflections across the coordinate axes are special cases where $c = -1$.

