DIGITAL REPRESENTATION OF ANALOG SIGNALS

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LECTURE OUTLINES

- Why Digital?!!
- Sampling and Reconstruction of Continuous Time Signals
- The Sampling Theorem
- Quantization of Continuous Amplitude Signals
- Coding Process

WHY DIGITAL?!

- Signals of our practical interests are analog (e.g. Radar and Voice Signals)
- Digital signals show more advantageous features among analog:
  - Systems of the digital form show more flexibility by means of system reconfiguration:

    Analog: the hardware must be redesigned, tested and verified in order to operate with jumping from one application to another.

    Digital: Reprogramming of digital systems is fair enough to reconfigure another type of applications.

  - Digital signals are more robust to channel noise and very easy to recover the original signal from the transmitted form.

WHY DIGITAL?!

- Working with digital is more accrue in determination of signal processor:

  Digital: Provides higher tolerance and control of accuracy requirements

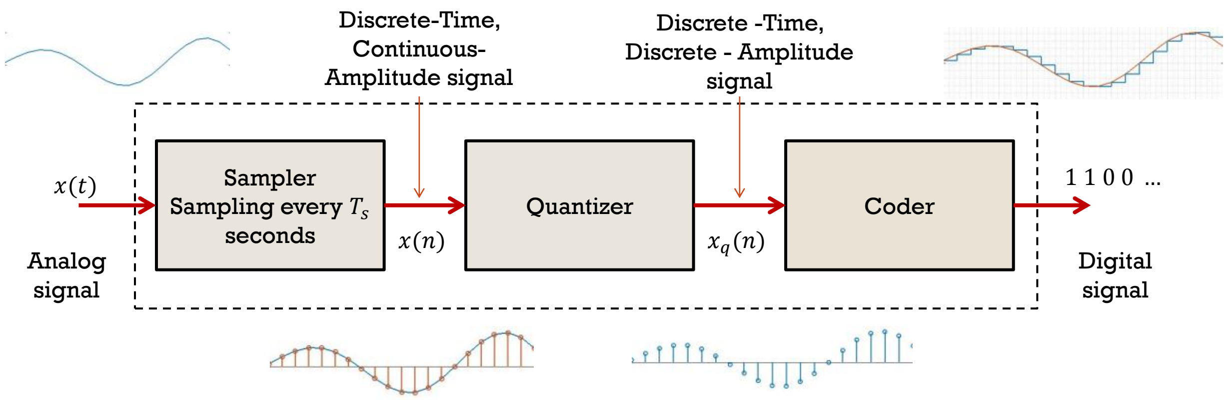
  Analog: In the designing process of system components, analog systems is very difficult to control the accuracy of analog signal processing

- Digital signals can be stored without loss, easy to transport between devices, and ability of processing signals remotely in Labs compared to analog signals.
- The implementation of hardware devices in digital systems are less expensive compared to analog devices.

DIGITAL SIGNAL GENERATION

- Signals such as voice, radar, audio and video are analog in their nature.
- To process analog signals in digital means, analog signals have to be pushed through three successive components which they are:
  - Sampling: The process of converting the analog (Continuous – Time, Continuous – Amplitude) signals into discrete – Time, Continuous – Amplitude.
  - Quantization: The process of converting the analog discrete – time, continuous – amplitude signals into discrete – time, discrete – amplitude (digital form) signals (Giving a finite set of values to the sampled form analog signal with high precision).
  - Coding: The process of representing the discrete amplitude signal by a binary (b – bits) sequence.

ANALOG – TO – DIGITAL BLOCKS



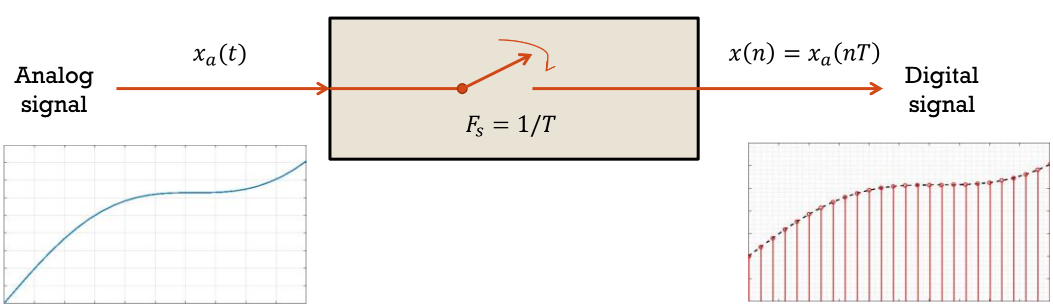
SAMPLING OF ANALOG SIGNAL

- There variety technique to sample the continuous – time (analog) signals, but our discussion focuses on ***periodic*** or ***uniform*** sampling.
- The sample process can be expressed by the relation:

$$x(n) = x_a(nT) \qquad -\infty < n < \infty$$

- Note that $x(n)$ is the discrete – time (sampled) signal of the original continuous – time (analog) signal sampled at every T seconds.
  - T is ***sample period*** or ***sample interval***.
  - $F_s = 1/T$, This term is called the ***sampling rate*** or ***Sampling Frequency (Hz)***

SAMPLING OF ANALOG SIGNAL



- The relation between the continuous – time and discrete – time variables is:

$$t = nT = \frac{n}{F_s}$$

SAMPLING OF ANALOG SIGNAL

- Consider the following CT signal:

$$x_a(t) = A\cos(2\pi F t + \theta)$$

- The sampled version of the signal at a sample rate of $F_s = 1/T$:

$$x_a(nT) = A\cos(2\pi F nT + \theta) = A\cos(\frac{2\pi F t}{F_s} + \theta)$$

- Comparing the above equation with the discrete time equation:

$$x(n) = A\cos(2\pi f n + \theta)$$

SAMPLING OF ANALOG SIGNAL

- We found a relation between the frequency variables (F) and (f):

$$f = \frac{F}{F_s}$$

- This relation is known as the normalized frequency which describe the frequency variable (f). This indicates that we can only specify the value of (F) in hertz if, and only if, F_s (sampling frequency) is known.
- We know that the range of (F): $-\infty < F < \infty$
- In discrete-time signals the situation is different: $-\frac{1}{2} < f < \frac{1}{2}$
- Substituting (F): $-\frac{1}{2T}\ or\ \frac{-F_s}{2} \leq\ F\ \leq \frac{1}{2T}\ or\ \frac{F_s}{2}$

SAMPLING OF ANALOG SIGNAL

- Observation:

It can be observed that there is a notable difference between continuous – time and discrete – time signals in their ranges. Uniform sampling of continuous-time signal implies a mapping of the infinite frequency range of variable (F) into a finite frequency range for the variable (f).

- The highest frequency in discrete-time signal is: $f = \frac{1}{2}$
- The highest value of (F) is: $F_{max} = \frac{F_S}{2} = \frac{1}{2T}$

- This introduce an ambiguity. It comes from the knowledge that the highest frequency in CT signals that can be distinguished when sampling the signal at a sample rate of (1/2T).

EXAMPLE

- Consider the following signals:

$$x_1(t) = \cos[2\pi\,(10)t] \quad \text{and} \quad x_2(t) = \cos[2\pi\,(50)t]$$

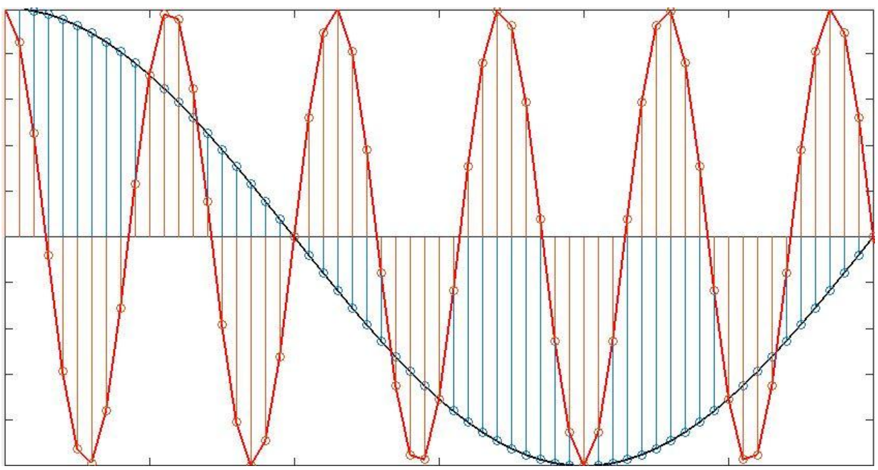
- We sampled both signals at a sample rate of $F_s = 40$ Hz, the resultant signals are:

$$x_1(t) = \cos\left[2\pi\,\left(\frac{10}{40}\right)n\right] = \cos\frac{\pi}{2}n \quad \text{and} \quad x_2(t) = \cos\left[2\pi\,\left(\frac{50}{40}\right)n\right] = \cos\frac{5\pi}{2}n$$

- Using identities: $\cos\frac{5\pi}{2}n = \cos\left(2\pi n + \frac{\pi n}{2}\right) = \cos\frac{\pi n}{2}$ **APROBLEM ARISED??**

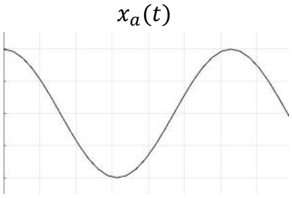
- Both sampled signals $x_1(n)$ and $x_2(n)$ are identical and indistinguishable fro each other. This relates to adding an ambiguity of whether this sample is belongs to $x_1(t)$ or $x_2(t)$. Since this ambiguity is raised, we say that the 50-Hz frequency is ***aliased*** with 10-Hz.

EXAMPLE OF ALIASING

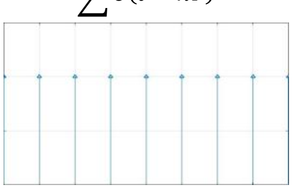


SAMPLING IN TIME - DOMAIN

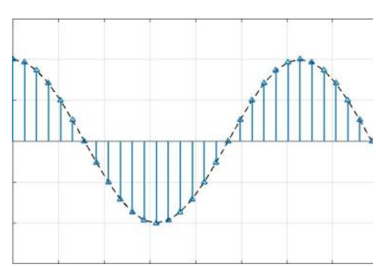
$x_a(t)$



$\sum \delta(t - nT)$

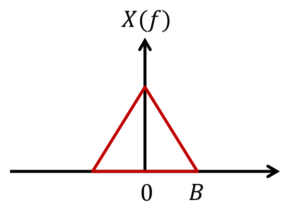


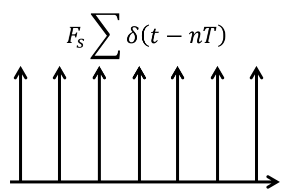
$x_a(n) = x_a(t) \sum \delta(t - nT)$

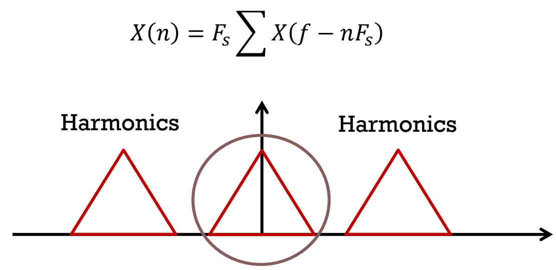


Sampling Period : T
Sampling Rate: $F_s = \frac{1}{T}$

SAMPLING IN FREQUENCY DOMAIN







How to recover the original signal?

THE SAMPLING THEOREM

- The ambiguity of signals due to sampling can be eliminated and the original signal reconstructed without any aliasing by selecting the sample rate $F_s = \frac{1}{T} = \frac{F_s}{2}$.
- Knowing that any signals above or below the sampling rate will result in aliasing problem and no further steps can be taken to reconstruct again.
- To avoid aliasing, we have to select a sampling rate sufficiently high. (Higher than the highest frequency components of the original signal.

$$F_s \geq 2F_{max} \quad \text{or} \quad F_s \geq 2B$$

- This condition ensures that all signals can be reconstructed without any ambiguity.

THE SAMPLING THEOREM

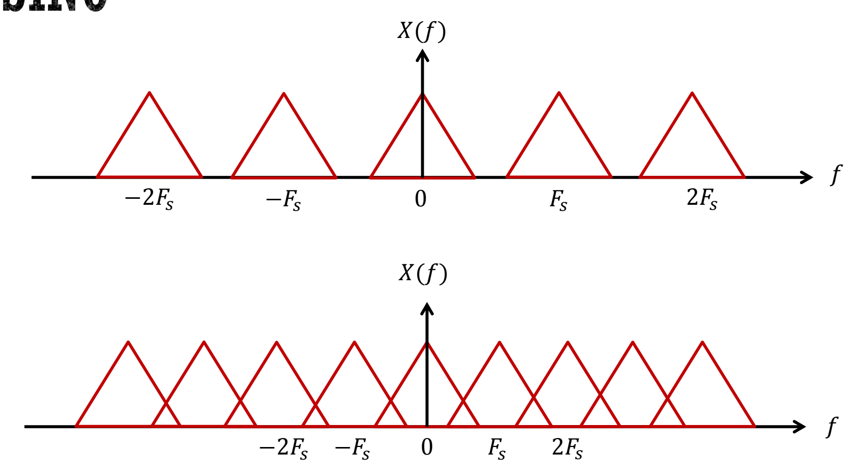
- The sampling theorem states that:

"If the highest frequency contained in an analog signal $x_a(t)$ is $F_{max} = B$ and the signal is sampled at a rate $F_s > 2F_{max} = 2B$, then $x_a(t)$ can be exactly recovered from its samples"

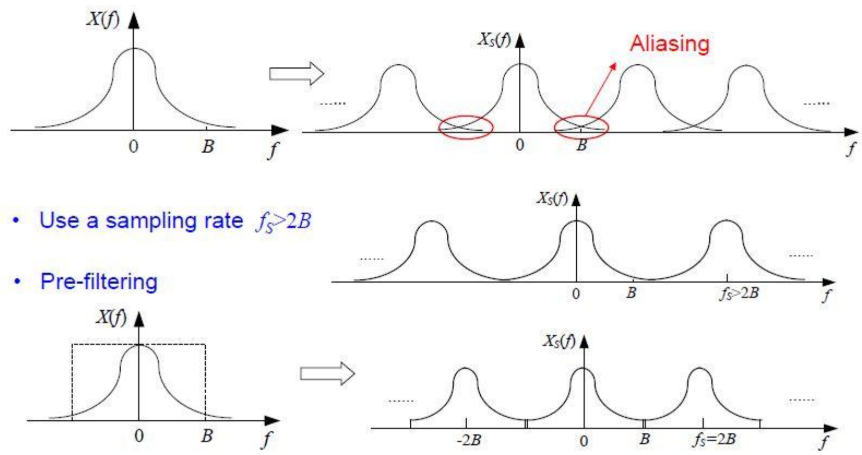
- If the condition is not satisfied, the aliasing problem will result in signal distortion. This is for ***band-limited*** signals

- What is the condition if the signal is not strictly band-limited??!

ALIASING



PRACTICAL CONSIDERATION



QUANTIZATION

- The process of converting the discrete – time, continuous – amplitude signal into a discrete – time, discrete – amplitude (fully digital) signal by expressing each sample with a finite set of values or digits (binary digits) is called ***quantization.***

- a companion with quantization process, non accurate representing of the continuous - amplitude of the quantized signals caused an error known as ***quantization error or quantization noise.***

- If we denote for the signals at the output port of the quantizer $Q[x(n)] = x_q(n)$, hence, the quantization error:

$$e_q(n) = x_q(n) - x(n)$$

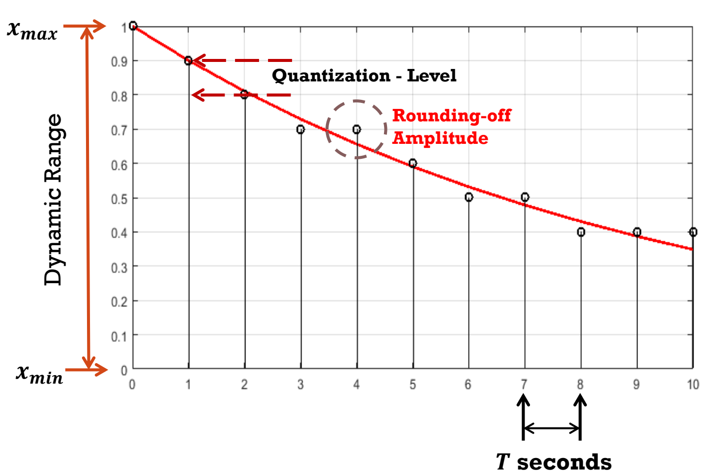
QUANTIZATION

- Suppose the following discrete – time signal:

$$x(n) = \begin{cases} 0.9^t & n \geq 0 \\ 0 & n < 0 \end{cases}$$

- The sampling has been accomplished with 1-Hz sampling frequency.
- We need n – digits to represents each samples with its binary value.

QUANTIZATION



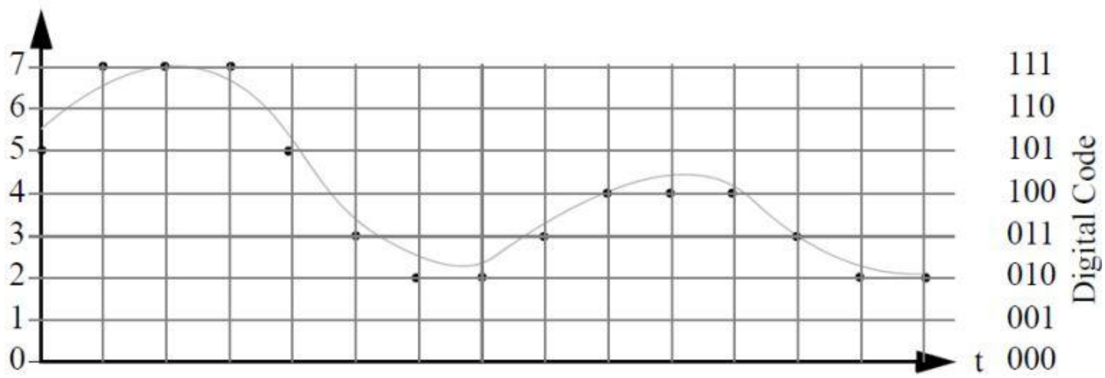
- The signal is rounded – off to the nearest value level.
- Each amplitude level, then, assigned to a digital code.
- The number of quantization levels can be obtained:

$$L = 2^b$$

- Each sampling samples is represented by (b) bits.
- The quantization region (dynamic range) is divided into (L) levels by:

$$\Delta = \frac{x_{max} - x_{min}}{L}$$

3-BITS CONVERSION



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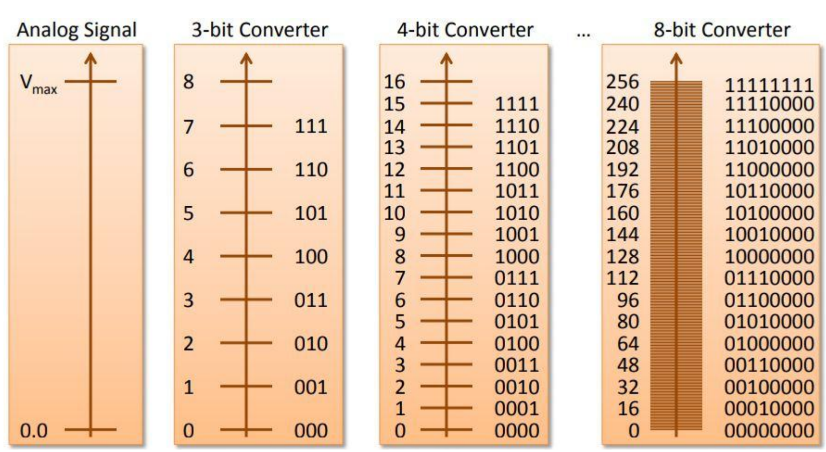
VARIOUS N-BITS QUANTIZATION

- The analog amplitude range may be split into different numbers of levels based on the $L = 2^b$, and the digital code is assigned to each level such as:



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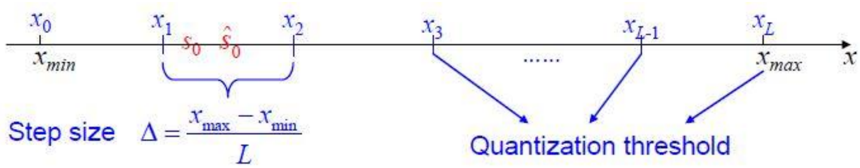
UNIFORM QUANTIZATION

- In uniform quantization, each quantized level is divided into L – levels ***EQUAL WIDTHS*** regions.



ROUND-OFF ERROR

- Round-off cannot be recovered
- As the number of bits increases, the amount of round-off error decreases.
- The application determines the number of bits to use in the ADC:
  - Telephone quality speech: 8 – bits
  - Music: 16 - bits